A Novel 6 dof Parallel Robot With Decoupled Translation and Rotation

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Abstract—This paper describes a new parallel robot with 6 degree of freedom (dof) and decoupled kinematics. The robot is composed of six legs with actuated prismatic joints; three legs control the spatial position of the center of the mobile platform, while the remaining legs are used to define the end effector orientation. The decouple kinematics is obtained by using a new approach for a triple spherical joint in conjunction with a 3-UPS parallel robot. In this work, the forward and inverse kinematic analysis of the mechanism is the first question answered, then, the velocity analysis is performed via screw theory, and, finally, a singularity and workspace analysis is performed using the previously obtained screw-based Jacobian.

Keywords: Decoupled Motion, Forward Kinematics, Six DoF Parallel Robot

I. Introduction

Parallel robots have been thoroughly studied during the last decades, in part due to their advantages when compared to serial robots, such as higher stiffness and precision [1]. Parallel robots with 6 dof can control the position and orientation of the mobile platform in tridimensional space. In this case, the most studied 6 dof parallel robot is the Stewart-Gough platform [2], which has been successfully applied in milling and positioning tools [3] [4], in supporting devices for surgical operations [5] [6], in flight simulators [7], and in field robots such as climbing or underwater robots, [8], [9]. In the last decades, several designs of 6 dof parallel robots have been proposed [1], but only a few of them have decoupled kinematics. Parallel robots with decoupled kinematics are those in which some actuators control a few pose parameters (for example the position of the platform) while the remaining actuators control the rest of pose parameters (for example the orientation). Some decoupled robots have been proposed by Innocenti [10], Bernier [11], Wenger [12], Zabalsa[13], Takeda [14], and Briotal [15]. There are three different types of decoupling [1]; the strong coupling, where each configuration parameter (for position and orientation) is a function of all joint variables, such as the Stewart-Gough platform; the complete decoupling, where each configuration parameter is a function of only one joint variable, for example the Orthoglide [12]; and the partial decoupling, where some configuration parameters are in function of only some joint variables, for example the Innocenti’s robot. Generally, decoupled parallel robots offer some advantages from the planning and control point of view, because of their relative simple forward kinematics and decoupled motion. In this paper we present a new parallel robot with six degree of freedom and decoupled kinematics, see Fig. 1. The robot is composed of six legs with actuated prismatic joints. Three legs control the position of one point of the mobile platform and the remaining legs control the orientation. This robot is cinematically equivalent to the architecture proposed by Innocenti [10].

The big difficulty for building a real robot based on the Innocenti’s kinematics design is the construction of a triple spherical joint. It is called a triple spherical joint to a kinematic union which allows four links to rotate around a common point. There have been previous work on the design of triple spherical joints such as [16], [17], [18], [11] and [14], among others. In [17] Zanganah and Angeles presented a redundant parallel robot, where six links connect at the center of the mechanism by means of a spherical joint and six clevis joints. Song et al., [18], presents a spherical joint for connecting three of more links. This spherical joint is made up of a ball and a spherical shell with holes that allows multiple links connect with the ball by means of sup-
port discs that are set within a gap between outer surface of the ball and inner surface of the spherical shell. In [11], Bernier et al. presented the Nabla 6, a decoupled parallel robot with six prismatic joints in the horizontal plane. In this robot pairs of prismatic joints share the same axis. Three legs are articulated on a triple spherical joint. The design of the spherical joint is based on the use of a central stack of rotating joints with a shared axis of rotation. The links are connected to this stack by means of a connecting link with two revolute joints that intersect at the stack central point. In [16], Bosscher and Ebert-Uphoff presented a mechanism for implementing multiple collocated spherical joints. This mechanism consists of two intermediate links with two rotational joints between the main links. The rotational joints axes intersect at a single point. In [14], Takeda et al. present a study in order to develop parallel robots with the Innocenti’s architecture. The triple spherical joint that they presented was composed of a small platform that is connected to each main link by two intermediate links with revolute joints whose axes were orthogonal. The platform also contains a spherical joint whose ball is connected to the mobile platform of the robot.

In this paper we propose a novel triple spherical joint. The design of this joint and the elements of the robot will be discussed in the following sections. In section III the kinematics analysis of the robot is presented. In section IV a study of the singularities and workspace is performed. Finally in section V we present the conclusions of this work.

II. System Description

The robot has six legs with actuated prismatic joints. Three internal legs control the position of the mobile platform while the remaining three external legs control the end effector orientation. The three internal legs are UPRRR kinematic chains (where U denotes an universal joint, P denotes an actuated prismatic joint, and R denotes a rotational joint). The external legs are UPS kinematic chains (where in this case S denotes a spherical joint). In order to obtain a decoupled kinematics, there must be a triple spherical joint between the three internal legs and the end effector. This is achieved by intersecting at a common point the axis of the last three rotational joints of each leg, see Fig. 2. In this case, the second rotational joint is orthogonal to both first and third rotational joints, thus all possible orientations of the legs with respect the mobile platform can be achieved (except those where there are singularities). On other hand, the robot’s end effector has its rotational axes arranged in a way that they intersect at a point, as illustrated in Fig. 3. When the three internal legs are assembled with the mobile platform, the intersection point of each leg will be coincident with the intersection point of the end effector, forming a triple spherical joint. Fig. 4 illustrates the equivalent kinematics scheme of the robot.

In order to verify the degrees of freedom of the parallel robot presented here, we use the Grüber - Kutzbach criterion [19],

$$F = \lambda(n - j - 1) + \sum \limits_{i} f_i$$  \hspace{1cm} (1)$$

where $F$ are the dof of the mechanism, $\lambda$ are the dof of the space in which the mechanism is intended to function, $n$ is the number of links in the mechanism, $j$ is the number of joints in the mechanism, $f_i$ are the degrees of relative motion allowed by joint $i$.

For the parallel robot we have the following values: $\lambda = 6, n = 20, j = 24, \sum f_i = 36$. Therefore, the degrees of freedom of the robot are $F = 6$. Consequently, using the length of the six prismatic joints as inputs, the position and orientation of the robot’s mobile platform can be defined.

III. Kinematics Analysis

In this section the direct and inverse kinematics of the proposed robot will be studied. The direct kinematics computes the orientation and position of the end effector from a known value of the lengths of the six actuators. On the other hand, the inverse kinematics computes the lengths of the six actuators from the actual position and orientation of the end effector. Lastly, the robot’s Jacobian is computed using the screw approach for the velocity analysis.
A. Direct Kinematics

The forward kinematics of this robot is solved in two parts. The first part will compute the position of the center of the end effector from the lengths of the three internal legs. The second part will compute the end effector orientation from the lengths of the three external legs.

A.1 Translational Problem

As observed in Fig. 5, the three internal legs form a pyramid shape where the vertex is the rotational center of the end effector. The following vectors are defined in the aforementioned figure.

\[
\mathbf{d} = \begin{bmatrix} d_x & d_y & d_z \end{bmatrix}^T
\]

related to the position of the triple spherical joint, \( \mathbf{a}_i = \begin{bmatrix} a_{i,x} & a_{i,y} & a_{i,z} \end{bmatrix}^T \), in connection to the position of the universal joint at the base body, and \( L_i \) related to the lengths of the pyramid edge. With these vectors, the total lengths of the first three actuators, \( \rho_i \), are,

\[
\rho_i = L_i - h_i
\]

where \( h_i \) is the distance between the lower universal joint and the upper pyramid vertex.

The internal length is computed with the following expression.

\[
L_i^2 = (d_x - a_{1,x})^2 + (d_y - a_{1,y})^2 + (d_z - a_{1,z})^2
\]

\[
L_i^2 = (d_x - a_{2,x})^2 + (d_y - a_{2,y})^2 + (d_z - a_{2,z})^2
\]

\[
L_i^2 = (d_x - a_{3,x})^2 + (d_y - a_{3,y})^2 + (d_z - a_{3,z})^2
\]

Solving for \( d_x \) and \( d_y \),

\[
d_x = \frac{k_1(1,2y) - k_2(1,2y)}{2k_3}
\]

\[
d_y = \frac{-k_1(1,2y) + k_2(1,2y)}{2k_3}
\]

Where \( k_i \) are defined with the following values,

\[
k_1 = L_1^2 - L_3^2 + a_{3,x}^2 - a_{1,x}^2 + a_{3,y}^2 - a_{1,y}^2
\]

\[
k_2 = L_2^2 - L_3^2 + a_{2,x}^2 - a_{1,x}^2 + a_{2,y}^2 - a_{1,y}^2
\]

\[
k_3 = a_{1,x}a_{2,y}a_{1,y}a_{2,x} + a_{3,x}(a_{1,y} - a_{2,y}) - a_{3,y}(a_{1,x} - a_{2,x})
\]

The coordinate \( z \) is obtained after replacing the values \( d_x \) and \( d_y \) in any of the equations (3).

A.2 Rotational Problem

Analyzing Fig. 6 may be noted that the length of each translational actuator is determined by the equation

\[
L_i = ||\mathbf{d} + \mathbf{R}(e_i - e) - b_i||
\]

where, \( \mathbf{R} \) is the end effector rotation matrix, \( \mathbf{e} \) is the eccentricity vector of the upper ring, \( e_i \) is the point of intersection of the actuator with the end effector and \( b_i \) is the point of intersection of each actuator with the base.

On the other hand, the square of the equation (7) is,

\[
L_i^2 = ||\mathbf{d} + \mathbf{R}(e_i - e) - b_i||^2
\]

which can also be expressed as,

\[
L_i^2 = (\mathbf{d} + \mathbf{R}(e_i - e) - b_i)^T(\mathbf{d} + \mathbf{R}(e_i - e) - b_i)
\]
Or in simplified form as,
\[
L_i^2 = d^T d + 2(d - b_i)^T R(c_i - e) - 2d^T b_i \\
+ (c_i - e)^T (c_i - e) + b_i^T b_i
\]
if subsequently the variables to calculate are placed on the left,
\[
2(d - b_i)^T R(c_i - e) = L_i^2 - d^T d + 2d^T b_i \\
- (c_i - e)^T (c_i - e) - b_i^T b_i
\]
and the following vectors are defined,
\[
\begin{align*}
\mathbf{u}_1 &= \begin{bmatrix} u_{x,i} & u_{y,i} & u_{z,i} \end{bmatrix}^T \\
\mathbf{v}_1 &= \begin{bmatrix} v_{x,i} & v_{y,i} & v_{z,i} \end{bmatrix}^T \\
\mathbf{w}_1 &= \begin{bmatrix} w_{x,i} & w_{y,i} & w_{z,i} \end{bmatrix}^T
\end{align*}
\]
\[
\begin{align*}
\mathbf{u}_1^T R \mathbf{v}_1 &= \mathbf{w}_1
\end{align*}
\]
We can appeal to quaternions, \( q \), for expressing the rotation matrix of the end effector, \( R \), and in this way finding a numerical solution to the equation (15).
\[
\begin{align*}
\mathbf{q} &= \begin{bmatrix} e_0 & e_1 & e_2 & e_3 \end{bmatrix}^T \\
1 &= e_0^2 + e_1^2 + e_2^2 + e_3^2
\end{align*}
\]
where \( R \) is defined by,
\[
\begin{bmatrix}
e_0^2 - e_1^2 - e_2^2 + e_3^2 & 2e_1e_2 - 2e_0e_3 \\
2e_0e_3 + 2e_1e_2 & e_0^2 - e_1^2 + e_2^2 - e_3^2 \\
2e_1e_3 - 2e_0e_2 & 2e_0e_1 + 2e_2e_3 \\
2e_0e_2 + 2e_1e_3 & 2e_2e_3 - 2e_0e_1
\end{bmatrix}
\]

To solve this nonlinear system, a vector equal to zero and associated with (15) \( y \) (17) will be defined,
\[
\begin{align*}
\phi &= \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_q \end{bmatrix}^T = \mathbf{0} \\
\phi_i &= \mathbf{u}_1^T R \mathbf{v}_1 - \mathbf{w}_1 \\
\phi_q &= \mathbf{q}^T \mathbf{q} - 1
\end{align*}
\]
with this expression, the numerical result for the four components of the quaternions, \( e_i \), will be obtained via the Newton-Raphson method,
\[
\mathbf{q}_{\text{next}} = \mathbf{q}_{\text{previous}} - \phi_q^{-1} \phi(\mathbf{q}_{\text{previous}})
\]
where \( \phi_q \) is the jacobian of the vector \( \phi \) and is defined by,
\[
\frac{\partial \phi}{\partial q} = \begin{bmatrix} \frac{\partial \phi_0}{\partial \phi_0} & \frac{\partial \phi_0}{\partial \phi_1} & \frac{\partial \phi_0}{\partial \phi_2} & \frac{\partial \phi_0}{\partial \phi_3} \\
\frac{\partial \phi_1}{\partial \phi_0} & \frac{\partial \phi_1}{\partial \phi_1} & \frac{\partial \phi_1}{\partial \phi_2} & \frac{\partial \phi_1}{\partial \phi_3} \\
\frac{\partial \phi_2}{\partial \phi_0} & \frac{\partial \phi_2}{\partial \phi_1} & \frac{\partial \phi_2}{\partial \phi_2} & \frac{\partial \phi_2}{\partial \phi_3} \\
\frac{\partial \phi_3}{\partial \phi_0} & \frac{\partial \phi_3}{\partial \phi_1} & \frac{\partial \phi_3}{\partial \phi_2} & \frac{\partial \phi_3}{\partial \phi_3} \\
2e_0 & 2e_1 & 2e_2 & 2e_3
\end{bmatrix}
\]
and each jacobian term, \( \frac{\partial \phi_j}{\partial \phi_i} \), is obtained after deriving \( \phi_i \),
\[
\phi_i = (v_{x,i}u_{x,i} + v_{y,i}u_{y,i} + v_{z,i}u_{z,i})e_0^2 \\
+ (v_{x,i}u_{x,i} + v_{y,i}u_{y,i} - v_{z,i}u_{z,i})e_1^2 \\
+ (v_{x,i}u_{x,i} - v_{y,i}u_{y,i} + v_{z,i}u_{z,i})e_2^2 \\
+ (2v_{x,i}u_{y,i} - 2v_{y,i}u_{x,i})e_0e_3 \\
+ (2v_{x,i}u_{z,i} + 2v_{y,i}u_{x,i})e_1e_2 \\
+ (2v_{x,i}u_{z,i} - 2v_{y,i}u_{x,i})e_1e_3 \\
+ (2v_{x,i}u_{z,i} + 2v_{y,i}u_{x,i})e_2e_3 - w_i = 0
\]

\section*{B. Inverse Kinematics}

If we define the position vector of end effector with respect to the base \( p \), the lengths of the six actuators are defined by the following equations,
\[
L_i = ||p + R\mathbf{c}_i - \mathbf{b}_i||, \forall i \in \{1,\ldots,3\} \quad (25)
\]
\[
L_j = ||p + R\mathbf{e}_j - \mathbf{a}_j||, \forall j \in \{4,\ldots,6\} \quad (26)
\]
Where \( e = [0\ 0\ e_z]^T \)

\section*{C. Velocity Analysis}

For each leg the resultant screw will be:
\[
\mathbf{s}_r = \omega_{1,i}\mathbf{b}_{1,1} + \omega_{2,i}\mathbf{b}_{2,1} + \mathbf{v}_i\mathbf{b}_{3,1} + \omega_{4,i}\mathbf{b}_{4,1} + \omega_{5,i}\mathbf{b}_{5,1} + \omega_{6,i}\mathbf{b}_{6,1}
\]

\section*{D. Screw Velocity Analysis
Where the jacobian, $J$, is,

$$ J = \begin{bmatrix}
  u_1^T & -u_1^T b_1 \\
  u_2^T & -u_2^T b_2 \\
  u_3^T & -u_3^T b_3 \\
  u_4^T & -u_4^T \tilde{e} \\
  u_5^T & -u_5^T \tilde{e} \\
  u_6^T & -u_6^T \tilde{e} 
\end{bmatrix} \quad (31) $$

$u_i$ are unit vector along the translational actuators, and $\tilde{e}$ is the matrix representation of cross product,

$$ \tilde{e} = \begin{bmatrix}
  -e_z \\
  e_y \\
  e_z \\
\end{bmatrix} = 
\begin{bmatrix}
  0 & -e_z & e_y \\
  e_y & 0 & -e_x \\
  -e_y & e_x & 0 
\end{bmatrix} \quad (32) $$

IV. Workspace and Singularities

The workspace of a parallel robot is divided into position workspace, which is defined as all the points reached by a specific orientation, and orientation workspace, which are all the possible rotations for a known position. The easy way to compute the workspace is using the lengths of the actuators; a point is outside of the robots workspace if the inverse kinematics yields values greater than those allowed by the hardware, otherwise the point will belong to the workspace, $L_{min} \leq L_i \leq L_{max}, \forall i = \{1, \ldots, 6\}$.

Singularities in a parallel robot are classified in inverse and direct singularities, and the usual way to compute them is through the velocity analysis, where the robot Jacobian is used,

$$ J_d \dot{q} = J_i \dot{x} $$

in this way, the direct singularities are associated with the singularities in the matrix $J_d$, and, accordingly, the inverse singularities with inverse Jacobian $J_i$. The most accepted methods to obtain a numerical value for singularities are the determinant and condition number of the direct and inverse Jacobians.

Figures 7 to 11 illustrate five workspaces and singularities for the proposed robot, similarly [20] describes the way how they were generated, this is a test volume is defined first, then the length of the translational actuator and the determinant of the robot Jacobian is calculated for each point within the defined test space, and finally, a red isosurface is created from all points where the actuator length has valid values, thus shaping the workspace. In a similar way, a yellow isosurface for singularities is build, but in this case, the points correspond to values where the determinant is equal to zero. Table I contains the robot dimensions used for computing the figures.

V. Conclusions

In this article, the kinematics of a new parallel robot with six degree of freedom has been explained and studied. The new robot has as its main characteristics the usage of a triple spherical joint and uncoupled kinematics. Despite the fact that cinematically the behavior of the robot is equivalent to a previously design proposed by Innocenti, the solution used in the triple spherical joint makes the robot a unique implementation, therefore a new mechanical design. Finally, the study of the workspace and singularities suggests that the new design may have some practical use in applications where a reduced workspace, but free of singularities, is required.
Fig. 9. Position Workspace for $R = \text{rot}_y \left(-\frac{\pi}{6}\right)$

Fig. 10. Position Workspace for $R = \text{rot}_x \left(\frac{\pi}{9}\right)$

Fig. 11. Position Workspace for $R = \text{rot}_y \left(-\frac{\pi}{9}\right)$

References