Stiffness Analysis of 6-RSS Parallel Manipulator

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Abstract—This paper describes a stiffness analysis of a 6-RSS parallel manipulator by using matrix structural analysis. The stiffness analysis is based on standard concepts of static elastic deformations. The formulation has been implemented in order to obtain the stiffness matrix that can be numerically computed by defining a suitable model of the manipulator, which takes into account the stiffness properties of each element such as links and joints. The obtained stiffness matrix was used to map the end-effector compliant displacements when external forces and torques are applied on it. Finally experimental test were performed to verify the proposed methodology.

Keywords: Parallel Manipulators, Stiffness Analysis, Matrix Structural Analysis

I. Introduction

Parallel manipulators typically consist of a moving platform that is connected to a fixed base by several serial chains, called limbs. Features of such systems can be better in stiffness and payload capacity with respect to the serial architectures, and have high velocity and acceleration during the operation. Furthermore, errors in the joints are not cumulative which contributes for its overall accuracy. Due to their characteristics they have been studied extensively both from theoretical and practical viewpoints. Prototypes have been conceived and built together with the development of theoretical investigations on kinematics and dynamics. The attention are focused to a number of possible applications such as manipulation [1, 2], packing and assembly/disassembly machines [3], motion simulation [4-7], milling machines [8], toys and sensors. However, they have some open problems such as small and complex workspace with internal singularities, stiffness and complexity of their direct kinematics [1, 9-11]. This paper deals with the study of stiffness of parallel manipulator.

Stiffness can be defined as the capacity of a mechanical system to sustain loads without excessive changes on its geometry [12, 13]. These produced changes on geometry, due to the applied forces, are known as deformations or compliant displacements.

Compliant displacements in a parallel robotic system produces negative effects on static and fatigue strength, wear resistance, efficiency (friction losses), accuracy, and dynamic stability (vibrations). The growing importance of high accuracy and dynamic performance for parallel robotic systems has increased the use of high strength materials and lightweight designs improving significant reduction of cross-sections and weight. Nevertheless, these solutions also increase structural deformations and may result in intense resonance and self-excited vibrations at high speed [13]. Therefore, the study of the stiffness becomes of primary importance to the design of multibody robotic systems in order to properly choose materials, component geometry, shape and size, and interaction of each component with others. Some examples of design procedures based on stiffness analysis can be found in [14-17].

The overall stiffness of a manipulator depends on several factors including the size and material used for links, the mechanical transmission mechanisms, actuators and the controller [12, 18]. In general, to realize a high stiffness mechanism, many parts should be large and heavy. However, to achieve high speed motion, these should be small and light. Moreover, one should point out that the stiffness is greatly affected by both the position and the values of the mechanical parameters of the structure parts [19].

There are three main methods to derive the stiffness model of parallel manipulators [20, 21]. These methods are based on the calculation of the Jacobian matrix [22, 23]; the Finite Element Analysis (FEA) [24-29] and the Matrix Structural Analysis (MSA) [20, 21, 30-36].

One of the first stiffness analysis has been conducted by Gosselin [9] and the stiffness of a Parallel Kinematic Machines is mapped onto its actuated joints whereas links are supposed as perfectly rigid. This approach was also applied by El-Khasawneh and Ferreira [37] and Zhang et al. [22]. To take into account the link flexibility, this first model was supplemented by the lumped model developed by Zhang [38]. Flexible links are then replaced by rigid beams mounted on revolute joints plus torsional spring located at the existing joints [18, 27, 39].

The uses of Finite Element Analysis models are reliable, but these models have to be remeshed over again which results in very tedious and time-consuming routines [40]. However these models are well adapted to validate analytical models or some experimental results

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[24-29, 37]. For example, Bouzgarrou et al. [24] used the Finite Element Analysis model to study static rigidity and natural frequencies of the T3R1 parallel robot.

Methods based on matrix structural analysis are simple and easy for computational implementation. Clinton et al. [41] used this approach to derive the stiffness matrix for each element of a Stewart platform and then assemble individual ones into a system stiffness matrix. This approach is also used in Huang et al. [40] to give the stiffness model of a machine frame considered as a substructure. The superposition principle is used to achieve the stiffness model of the machine structure as a whole.

In this paper is presented a methodology to obtain the global stiffness matrix of a parallel manipulator, using the matrix structural analysis, and known the stiffness matrix of each structure element. With the stiffness matrix one do the map of the compliant displacement of end-effector as function of the external efforts applied on its center, and the kinematic model of parallel manipulator. The methodology is applied to a 6-RSS parallel manipulator and the results are compared with those obtained from experimental tests.

II. Matrix Structural Analysis

In this paper, the stiffness matrix is obtained from the method Matrix Structural Analysis (MSA), also known as the displacement method or direct stiffness method (DSM). The methods of structural analysis is based on the idea of breaking up a complicated system into component parts, discrete structural elements, with simple elastic and dynamic properties that can be readily expressed in a matrix form. The discrete structure is composed by elements which are joined by connecting nodes. When the structure is loaded each node suffers translations and/or rotations, which depend on the configuration of the structure and the boundary conditions. For example, in a fixed linkage no displacement occurs. The nodal displacement can be found from a complete analysis of the structure. The matrices representing the beam and the joint are considered as building blocks which, when fitted together in accordance with a set of rules derived from the theory of elasticity, provide the static and dynamic properties of the whole structure [30].

A. Stiffness of joint and beam

The stiffness of a joint is given by [33]:

\[ k_{j} = \begin{bmatrix} k_{b} & -k_{b} \\ -k_{b} & k_{b} \end{bmatrix} \]  

where \( k_{b} \) is given by

\[ k_{b} = \begin{bmatrix} \frac{A_{j}E_{j}}{L_{j}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12E_{j}I_{zj}}{L_{j}^{3}} & 0 & 0 & \frac{6E_{j}I_{zj}}{L_{j}^{2}} & 0 \\ 0 & 0 & \frac{12E_{j}I_{yj}}{L_{j}^{3}} & 0 & -\frac{6E_{j}I_{yj}}{L_{j}^{2}} & 0 \\ 0 & 0 & 0 & G_{j}L_{j} & 0 & 0 \\ 0 & 0 & -\frac{6E_{j}I_{yj}}{L_{j}^{2}} & 0 & 4E_{j}I_{yj} & 0 \\ 0 & 0 & \frac{6E_{j}I_{zj}}{L_{j}^{2}} & 0 & 0 & \frac{4E_{j}I_{zj}}{L_{j}} \end{bmatrix} \]  

In Equation (3) \( E_{j} \) and \( G_{j} \) are the modulus of elasticity and the shear modulus of element \( j \) respectively; \( L_{j} \) is the beam length, \( I_{yj} \), \( I_{zj} \) are the moment of areas about the \( Y \) and \( Z \) axes, respectively. \( J_{j} \) is the Saint-Venant torsion constant and \( A_{j} \) is the cross-sectional area.

For application of MSA is necessary to write the stiffness matrices of all elements in the same reference frame. This reference transformation must be computed for all elements before the assembly of the stiffness matrix of the structure. The transformation matrix, \( T_{p} \), can be obtained from linear algebra [42].

Thus, the stiffness matrix of the elements in a common reference frame (elementary stiffness matrix), for segments, \( k_{j}^{e} \), and for joints, \( k_{j}^{e} \), can be expressed as

\[ [k_{j}^{e}] = [T_{p}][k_{j}][T_{p}]^{T} \]  

\[ [k_{j}^{e}] = [T_{p}][k_{j}][T_{p}]^{T} \]  

After obtaining the stiffness matrix of beams and joints in a common reference frame, the stiffness matrix of a structure can be obtained by using the MSA. Based on how the structure elements are connected, through their nodes, it is possible to define a connectivity matrix. Since segment and joint stiffness are known, the global stiffness matrix can be obtained by a superposition procedure. The described methodology to obtain the global stiffness matrix considers the structure as free, i.e., without motion constraints. By applying boundary conditions when the displacements are known, a new invertible stiffness matrix \( K \) can be obtained, and then the compliant displacement can be computed as

\[ [U] = K^{-1} [W] \]

Where \( U \) are the compliant displacements and \( W \) are the applied external wrenches.
Although the FEA and MSA have the same basic equations, Eqs. (2), (3), (4) and (6), one can point out some advantages of the MSA method: (a) A robotic structure is composed by segments and joints. Then, each segment and each joint can be modeled by only two nodes for the MSA analysis. Otherwise on the FEA each beam is divided in several nodes and the joints stiffness, in general, are not considered. (b) Using a commercial FEA software (the usual procedure) one does not have the control of the solver. Otherwise, in the MSA method the stiffness matrix assembling can be follow step-by-step. (c) In the FEA method at each changing of the structure configuration a remeshed must be made, increasing the computational cost. In the MSA method is only necessary to improve the inverse kinematic model to obtain the stiffness mapping for all structure configurations.

III. The 6-RSS Parallel Manipulator

The 6-RSS parallel manipulator is a 6 degree of freedom manipulator, which is characterized by a base and a mobile platform, connected by six RS-SS segments, where the R-joint is on the Cartesian axes (points bi), two joints by axis, as shown in Fig. 1. The Cartesian system is the base of the structure where the actuators are mounted, reducing dead loads. The links can be constructed by light and resistant materials, with low inertia and in a modular construction. One extremity of SS-segments is connected to the platform, consisting of a virtual cube, where the S-joints are tied on the center of its faces. Kinematics variables are the input angles αi (i=1 to 6) of the R-joints. The studied structure has the RS-segments and the SS-segments the same length and |b1b2| = |b2b4| = |b4b6| = |p1p2| = |p3p4| = |p5p6|.

Figure 1. Generic configuration 6-RSS parallel manipulator.

Figure 2 shows the prototype constructed at the Laboratory of Automation and Robotics in Uberlândia/Brazil.

Figure 2. The built prototype of a 6-RSS parallel manipulator.

Kinematic equations can be obtained by using a suitable analysis procedure with a vector and matrix formulation [43]. Equations can be written by considering a generic reference frame fixed to the base (θb, x_F, y_F, z_F) and another at the platform (θ_M, x_M, y_M, z_M) as shown in Fig. 3. The parameters are: αi are the generalized coordinates; bi the position vector of the base points bi (position of the R-joint in the base), Si and Li the arm and the forearm vectors, all referred to the base reference frame, and Si and Li their lengths; pi the position vector of the platform points pi (position of the S-joint at the mobile platform), referred to the mobile reference frame. All parameters for i=1 to 6.

Figure 3. Vectors for modeling the 6-RSS structure.
From Figure 3, for i= 1 to 6, one can obtain:

\[ L_i = [R]p_i + t - b_i - S_i \]  \hspace{1cm} (7)

where

\[ b_i = (b_{ix}, b_{iy}, b_{iz}) \]  \hspace{1cm} (8)

\[ p_i = (p_{ix}, p_{iy}, p_{iz}) \]  \hspace{1cm} (9)

\[ S_i = (S_{ix}, S_{iy}, S_{iz}) \]  \hspace{1cm} (10)

\[ L_i = (l_{ix}, l_{iy}, l_{iz}) \]  \hspace{1cm} (11)

\[ S_i = (S_i^2 + S_i^2 + S_i^2)^{1/2} \]  \hspace{1cm} (12)

\[ L_i = (l_{ix}^2 + l_{iy}^2 + l_{iz}^2)^{1/2} \]  \hspace{1cm} (13)

\[
[R] = \begin{bmatrix}
    n_x & a_x & a_x \\
    n_y & a_y & a_y \\
    n_z & a_z & a_z \\
\end{bmatrix}
\]  \hspace{1cm} (14)

In Equation (7) only the lengths \( L_i \) of the vector \( L_i \) are known. Then, one can write:

\[
L_i^2 = ([R]p_i + t - b_i - S_i)^2 \]  \hspace{1cm} (15)

Developing Eq. (15), one have:

\[
2p_{ix}(n_xl_{ix} + n_yl_{iy} + n_zl_{iz} - n_xS_{ix} - n_yS_{iy} - n_zS_{iz}) + \\
2p_{iy}(n_yl_{ix} + n_xl_{iy} + n_zl_{iz} - n_xS_{ix} - n_yS_{iy} - n_zS_{iz}) + \\
2p_{iz}(n_zl_{ix} + n_xl_{iy} + n_yl_{iz} - n_xS_{ix} - n_yS_{iy} - n_zS_{iz}) + \\
2b_{ix}(S_{ix} - l_{ix} - n_xp_{ix} - n_yp_{iy} - n_zp_{iz}) + \\
2b_{iy}(S_{iy} - l_{iy} - n_xp_{ix} - n_yp_{iy} - n_zp_{iz}) + \\
2b_{iz}(S_{iz} - l_{iz} - n_xp_{ix} - n_yp_{iy} - n_zp_{iz}) + \\
\left( \alpha_1^2 + \alpha_2^2 + \alpha_3^2 \right) + p_{ix}^2 + p_{iy}^2 + p_{iz}^2 + b_{ix}^2 + b_{iy}^2 + b_{iz}^2 - \\
2(t_{ix}S_{ix} + t_{iy}S_{iy} + t_{iz}S_{iz}) - L_i^2 = 0 \]  \hspace{1cm} (16)

Choosing proper reference frames, Eq. (16) can be simplified. In this work the used reference frames are: \((X_b, Y_b, Z_b)\) at the base, where the origin coincides with point \( b_1 \); and \((X_p, Y_p, Z_p)\) fixed at the point \( p_1 \) of mobile platform as represented in Fig. 1. The rotation matrix \([R]\) is given by the Euler angles (rotation \( \phi_i \) about \( x \), rotation \( \phi_i \) about \( y \) and rotation \( \phi_i \) about \( z \)), where \( s \) and \( c \) are the abbreviation of sine and cosine respectively:

\[
R = \begin{bmatrix}
    c\phi_2c\phi_3 & -c\phi_2s\phi_3 & s\phi_2 \\
    s\phi_2c\phi_3 + c\phi_3s\phi_2 & -s\phi_2c\phi_3 + s\phi_3c\phi_2 & -c\phi_3 \\
    -c\phi_3s\phi_2 - s\phi_2c\phi_3 & c\phi_2s\phi_3 - s\phi_2c\phi_3 & c\phi_3c\phi_2 \\
\end{bmatrix} \]  \hspace{1cm} (17)

The inverse kinematic model of the 6-RSS parallel manipulator obtains the co-ordinates \( \alpha_i \) (i=1 to 6) when the position \((t_x, t_y, t_z)\) and orientation \((\phi_{ix}, \phi_{iy}, \phi_{iz})\) of the platform are known. Equation (16) constitute a system with six variables: \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 \) and \( \alpha_6 \), representing the inverse kinematic model of the 6-RSS parallel manipulator. In this system, vectors \( p_i, b_i \) and \( t_i \), the matrix \([R]\) and lengths of \( S_i \) and \( L_i \) vectors are known.

Only the generalized coordinates \( \alpha_i \) (i=1 to 6) are unknown. This system has 64 solutions in general. But the system solution has repeated roots, including configurations not possible to be attained by the mechanism, having then only one set of roots \( \alpha_i \) (i=1 to 6) valid [43, 44].

IV. Stiffness Model of a 6-RSS Parallel Manipulator

In this paper are presented results of the stiffness analysis considering only the compliance of segments. The arms and forearms are modeled as beam, Fig. 4 and the stiffness of whole structure is carried out according to the elements are arranged in the structure, Fig. 5.
The following parameters were used in the model: length of the arms and forearms equal to 0.3m; \( |b_1b_2| = \frac{d}{2} \) and \( |b_2b_3| = \frac{d}{2} \); the segments are built with steel (\( G = 0.8 \times 10^{11} \text{ N/m}^2 \)); the cross-sectional area is circular with 0.005m diameter. The boundary conditions are given by actuators considered as blocked and, therefore, the forearms can be considered as fixed in the rotational joints (nodes 1, 7, 8, 13, 14 and 19). The external force and torque are applied on node 4, which is the center of the end-effector.

From nodes, defined on Fig. 5, the structure has 19 nodes with 6 d.o.f. at each node. Then the system has 114 d.o.f. with a 18x12 connectivity matrix. The stiffness matrix of the structure after considering the boundary conditions (six fixed rotational joints) becomes a 78x78 matrix \([33]\).

In order to verify the presented analysis using the MSA method, a model was built in FEA using commercial software. The used system is a PC Pentium Dual Core®, 2 Gb of memory, and the Ansys® software.

Tables 1 and 2 provides results obtained from the FEA and MSA models. In Tables 1, 2 and 3, the used element is a beam type divided in 10 parts.

Results of Numerical simulations presented in Tables 1 and 2 were made in order to show the soundness of the model due to symmetry of the structure. One can note that the compliant displacements are the same in the direction of the applied forces when other forces and torques are zero and, if the same effort values are applied, the compliant displacements are equals. In Table 3 are presented the compliant displacements results for a generic configuration of the 6-RSS structure.

Table 1. Compliant displacements of node 4 for \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0^\circ \). Forces are given in [N] and torques in [Nm].

<table>
<thead>
<tr>
<th>U</th>
<th>( F_x=(0,0,0) )</th>
<th>( T_x=(0,0,0) )</th>
<th>( F_x=(0,10,10) )</th>
<th>( T_x=(0,0,0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_x ) [m]</td>
<td>MSA</td>
<td>0.002168</td>
<td>-0.000532</td>
<td>0.001103</td>
</tr>
<tr>
<td>FEA</td>
<td>0.002173</td>
<td>-0.000534</td>
<td>0.001105</td>
<td></td>
</tr>
<tr>
<td>( \delta_y ) [m]</td>
<td>MSA</td>
<td>-0.000532</td>
<td>0.002168</td>
<td>0.001103</td>
</tr>
<tr>
<td>FEA</td>
<td>-0.000534</td>
<td>0.002173</td>
<td>0.001105</td>
<td></td>
</tr>
<tr>
<td>( \delta_z ) [m]</td>
<td>MSA</td>
<td>-0.000532</td>
<td>-0.000532</td>
<td>0.001103</td>
</tr>
<tr>
<td>FEA</td>
<td>-0.000534</td>
<td>-0.000534</td>
<td>0.001105</td>
<td></td>
</tr>
<tr>
<td>( \delta\phi_x ) [rad]</td>
<td>MSA</td>
<td>0.000857</td>
<td>-0.005384</td>
<td>0.001103</td>
</tr>
<tr>
<td>FEA</td>
<td>0.000854</td>
<td>-0.005378</td>
<td>0.001105</td>
<td></td>
</tr>
<tr>
<td>( \delta\phi_y ) [rad]</td>
<td>MSA</td>
<td>0.001701</td>
<td>0.000857</td>
<td>0.001103</td>
</tr>
<tr>
<td>FEA</td>
<td>0.001727</td>
<td>0.000854</td>
<td>0.001105</td>
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<tr>
<td>( \delta\phi_z ) [rad]</td>
<td>MSA</td>
<td>-0.005384</td>
<td>-0.000534</td>
<td>0.001103</td>
</tr>
<tr>
<td>FEA</td>
<td>-0.005378</td>
<td>-0.000534</td>
<td>0.001105</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Compliant displacements of node 4 for \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0^\circ \). Forces are given in [N] and torques in [Nm].

<table>
<thead>
<tr>
<th>U</th>
<th>( F_x=(0,0,10) )</th>
<th>( T_x=(0,0,0) )</th>
<th>( F_x=(10,10,10) )</th>
<th>( T_x=(0,0,0) )</th>
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<tr>
<td>( \delta_x ) [m]</td>
<td>MSA</td>
<td>-0.000532</td>
<td>0.001103</td>
<td>0.000000</td>
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<td>MSA</td>
<td>0.002168</td>
<td>0.001103</td>
<td>0.000000</td>
</tr>
<tr>
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<td>0.002173</td>
<td>0.001105</td>
<td>0.000000</td>
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<tr>
<td>( \delta\phi_x ) [rad]</td>
<td>MSA</td>
<td>0.001701</td>
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<td>0.000000</td>
</tr>
<tr>
<td>FEA</td>
<td>0.001727</td>
<td>0.001105</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>( \delta\phi_y ) [rad]</td>
<td>MSA</td>
<td>-0.005384</td>
<td>-0.002826</td>
<td>0.000000</td>
</tr>
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<td>FEA</td>
<td>-0.005378</td>
<td>-0.002796</td>
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<tr>
<td>( \delta\phi_z ) [rad]</td>
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<td>0.000000</td>
</tr>
<tr>
<td>FEA</td>
<td>0.000854</td>
<td>-0.002796</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Compliant displacements of node 4 for \( \alpha_1 = 1.5^\circ; \alpha_2 = 5^\circ; \alpha_3 = 5^\circ; \alpha_4 = 3.5^\circ; \alpha_5 = 3^\circ; \alpha_6 = 1^\circ \) using the MSA. Forces are given in [N] and torques in [Nm].

<table>
<thead>
<tr>
<th>U [MSA]</th>
<th>( F_x=(10,0,0) )</th>
<th>( T_x=(0,0,0) )</th>
<th>( F_x=(10,10,10) )</th>
<th>( T_x=(0,0,0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_x ) [m]</td>
<td>0.002366</td>
<td>-0.000456</td>
<td>0.000763</td>
<td>0.000763</td>
</tr>
<tr>
<td>( \delta_y ) [m]</td>
<td>-0.001190</td>
<td>0.002302</td>
<td>0.000650</td>
<td>0.000650</td>
</tr>
<tr>
<td>( \delta_z ) [m]</td>
<td>-0.000391</td>
<td>-0.001196</td>
<td>0.000696</td>
<td>0.000696</td>
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<tr>
<td>( \delta\phi_x ) [rad]</td>
<td>0.008324</td>
<td>-0.005218</td>
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</tr>
<tr>
<td>( \delta\phi_y ) [rad]</td>
<td>0.002119</td>
<td>0.008197</td>
<td>0.005179</td>
<td>0.005179</td>
</tr>
<tr>
<td>( \delta\phi_z ) [rad]</td>
<td>-0.005466</td>
<td>0.002205</td>
<td>0.004672</td>
<td>0.004672</td>
</tr>
</tbody>
</table>

It was built a prototype of the 6-RSS not actuated, Fig. 6, to perform experimental tests. The blocked actuators (obtained using bolts and nuts) are used as boundary conditions (nodes 1, 7, 8, 13, 14 and 19) and then, the rotational joints can be considered as fixed.

Three different loads in the center of platform, point 4, had been applied. The values of mass are: \( M_1 = 0.9935\text{kg}; M_2 = 1.2435\text{kg} \) and \( M_3 = 1.4915\text{kg} \). The acceleration of gravity is \( g = 9.81 \text{m/s}^2 \).

Compliant displacements in the \( Z_6 \) direction, in points 3, 5, 11 and 17, were obtained with a dial indicator with a resolution of 1µm. The compliant displacements were obtained from a statistical planning to determine the number of required measurements, where results are shown in Table 4.

From Table 4 the results prove the validity of the MSA method with errors less than 9 percent. Next step of this work will consider the compliance of joints and the
experimental tests for measuring all compliant displacements: \( \delta_x, \delta_y, \delta_z, \delta \phi_x, \delta \phi_y \) and \( \delta \phi_z \).

V. Conclusions

In this paper was presented a methodology to obtain the stiffness matrix for robotic structures by using the matrix structural analysis (MSA).

The stiffness matrix enables to obtain the mobile platform displacement when an external wrench is applied.

For application of the presented methodology the kinematic model of the structure and the stiffness matrices of its elements (segments and joints) must be known.

The great advantage of the matrix structural analysis method to obtain the stiffness matrix of a whole structure is not to require calculating the Jacobian of the structure and, therefore, does not need to work with differential equations. Another advantage is that the method enables to perform the mapping of compliant displacements for nodes related to different configurations of the structure.

The methodology was applied in a complex parallel structure 6-RSS and, in order to simplify the model, it was considered only the segments stiffness. The blocked actuators are used as boundary conditions and then, the rotational joints can be considered as fixed. To evaluate the method, first the numerical results were compared with a model built and solve by a commercial finite element program.

Experimental tests were presented showing the validity of the MSA method with errors less than 9 percent.

The methodology presented can be applied to analyze the stiffness of other robots structures.

Future works concern now in improving the stiffness model considering the compliance of the joints and develop a measurement device to obtain all linear and angular compliant displacements.

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References


