Pose Accuracy Analysis of Master Slave Surgical Robot System

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Abstract—The Position and orientation accuracy of robot manipulator has long become a major issue to be considered in its advanced application. The simplified expressions for the differential vector of the end-effector have been proposed based on the original expressions derived by Veitschegger and Wu in 1986. According to this new expression, a linear error model that described the end-effector position and orientation errors of the master slave surgical robot system due to kinematics parameters errors has been presented. A computer program to perform the accuracy analysis has been developed in MATLAB. This methodology and software are applied to the accuracy analysis of a master-slave surgical robot system. The position error in its workspace cross section (XOZ) has been plotted as 3D surface graph and discussed.

Keywords: master slave surgical robot, positioning accuracy, workspace mapping, error model

I. Introduction

The first robotic surgery was done in 1985 when a PUMA robotic surgical arm used by Kwoh for a neurosurgical biopsy. As a promising new technology, Robotic surgery is now becoming the medical mainstream, and has been applied to many different surgical disciplines [1]. The master-slave surgical robot system is a more effective surgical robotic system, the slave manipulator is the actual robot manipulator that performs the surgery within the patient’s body while the master is an interface device that allows the surgeons to control the slave [2]. The daVinci surgical system is the most prominent commercially master-slave surgical robot system. Accuracy is the most important characteristic of surgical robots, which inherently determines its applicability, functionality and safety [3]. As the first neurosurgical robot, the application accuracy of the NeuroMate was 0.95 ± 0.44 mm (mean ± standard deviation). In case of Retinal microsurgery, the positioning accuracy is desired to be less than 10 microns, the absolute positioning precision of Steady-hand in surgery is approximately 5 microns. In orthopedic surgical field, ROBODOC has a 0.5 mm intrinsic accuracy and a 1.2 mm application accuracy in average. As the most successful Laparoscopy surgery robot, the intrinsic accuracy of the daVinci was 1.35 mm TRE with FLE (the fiducial localization error) 1.02 ± 0.58 mm. Studies on accuracy analysis of the surgical robot are great valuable in its applications [4].

Considerable research has been performed on the topics of kinematics error model, error analysis, and calibration of single robot manipulator to improve the positioning accuracy [5]. Many kinematics error models have been developed for subsequent use in the error analysis and kinematics parameters identification of robot manipulators. Hayati [6] introduced an extra rotation Rot(y, β) to the Denavit and Hartenberg (D-H) model to represent small errors in the case of two adjacent parallel joints or nearly parallel joints, and proposed a linear model which relates the parameter errors to the end-effector position error of the serial manipulator. Based on the kinematics model modified by Hayati, Veitschegger and Wu [7] developed a linear model as a function of five kinematics parameter errors for each link, which includes the second order error terms. Zhuang et al [8] gave a comprehensive bibliography on kinematics modeling, and proposed a complete and parametrically continuous model suitable for the construction of error models by adopting a singularity-free line representation. According to the CPC model, a linear error model can be constructed in which all error parameters are independent and span the entire geometric error space. The error models mentioned above were only for single robot, the error model for the master slave robot system has not been reported in literature.

In this paper, according to the Khalil and Kleinfinger notation, the expressions of the differential translation vector and the differential rotation vector of the end-effector presented by Veitschegger and Wu have been simplified. The position and orientation error model of the master slaver surgical robot has been present based on this new expression. To perform the accuracy analysis on the master salve robot system manipulator, a computer program has been developed in MATLAB, a simulation was applied to a master slave surgical robot system. The position error of the master slave surgical robot system in its workspace cross section (XOZ) has been evaluated by nominal kinematics parameters and range of kinematics parameters errors, the error distribution has been plotted as 3D surface graphs and discussed.

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II. Workspace mapping

A. Kinematics model

The robot kinematics model is based on the Khalil and Kleinfinger notation which is derived from the well-known D-H notation [9]. The Khalil and Kleinfinger notation has four parameters, such as the link twist angle $\alpha_i$, the link length $a_i$, the joint angle $\theta_i$, the axis shift $d_i$, to model the translation and rotation of coordinates between successive reference systems, as shown in Figure 1. In order to obtain a parametrically continuous model in the case of two adjacent parallel joints or nearly parallel joints, an additional rotation term $\beta_i$ about the y-axis known as a Hayati parameter is introduced as the fifth parameter. The translation and rotation of one joint coordinate frame relative to the preceding joint coordinate frame can be described by a homogenous transformation matrix, which is a function of five kinematics parameters. The homogenous transformation matrix is given as:

$$
A_i = \text{Rot}(y, \beta_i)\text{Rot}(x, \alpha_i)\text{Trans}(x, a_i)\text{Rot}(z, \theta_i)\text{Trans}(z, d_i)
$$

$$
= \begin{bmatrix}
    c_i c_{\beta_i} + s_i s_{\alpha_i} s_{\beta_i} & c_i s_{\alpha_i} s_{\beta_i} - s_i c_{\beta_i} & c_i s_{\beta_i} & d_i c_{\alpha_i} s_{\beta_i} + a_i c_{\beta_i} \\
    s_i c_{\alpha_i} c_{\beta_i} & c_i c_{\alpha_i} & -s_i c_{\alpha_i} & d_i c_{\alpha_i} \\
    s_i s_{\alpha_i} c_{\beta_i} - c_i s_{\beta_i} & c_i s_{\alpha_i} c_{\beta_i} + s_i s_{\beta_i} & c_i c_{\beta_i} & d_i c_{\alpha_i} c_{\beta_i} - a_i s_{\beta_i} \\
    0 & 0 & 0 & 1
\end{bmatrix}
$$

(1)

where $s_i$ and $c_i$ are used as abbreviations for $\sin \theta_i$ and $\cos \theta_i$, respectively.

For an N degrees-of-freedom robot manipulator, the position and orientation of the end-effector frame relative to the robot base frame can be represented by the equation:

$$
T_N = A_1 A_2 \cdots A_N
$$

(2)

B. Mapping relationship

As the master manipulator and the slave manipulator in the surgical robot system commonly have different structure and kinematics, so the workspace differences between the master and the slave should be considered in the mapping process.

To meet the mapping conditions, the valid workspace of the master manipulator is defined as a volume within which each point can be mapped to a correspondent point in the slave manipulator workspace, which is only a subset of the reachable workspace of the master manipulator.

To describe the proportion of the valid workspace of the master manipulator with relation to its reachable workspace, the workspace mapping quantity $Q$ is calculated as:

$$
Q = \frac{W_v}{W_r}
$$

(3)

where: $W_v$ is the valid workspace of the master manipulator, $W_r$ is the reachable workspace of the master manipulator.

For most robots with many DOF and arbitrary joint ranges, analytic formulation of the workspace can’t be performed, the appropriate value of the workspace mapping quantity $Q$ can be calculated by using the Monte Carlo method.

A large number of points are randomly distributed in the reachable workspace of the master manipulator, and for each of the points, the inverse kinematics of the slave manipulator is solved, if the real solution of the joint variables exists, the point is viewed as in the valid workspace of the master manipulator. The following expression may be computed to evaluate the workspace mapping quantity:

$$
Q_N = \frac{N_v}{N_r}
$$

(4)

where: $N_r$ is the number of Monte Carlo points in the master manipulator’s reachable workspace, $N_v$ is the number of points in the valid workspace of the master manipulator.

The linear mapping relationship between the master manipulator and the slave manipulator in the workspace can be represented by the following equations [10]:

$$
P_s = R P_m + \Delta_p
$$

(5)

$$
\Delta_s = R \Delta_m + \Delta_0
$$

(6)

where $R$ is the Rotational matrix between the base coordinates of the master manipulator and the base coordinate of the slave manipulator.
\[ R = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}, \]

K is the Proportion matrix,

\[ K = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}, \]

\[ P_s = \left[ P_{sx} \ P_{sy} \ P_{sz} \right]^T \]

\[ P_m = \left[ P_{mx} \ P_{my} \ P_{mz} \right]^T \]

\[ \Delta_s = \left[ \Delta_{sx} \ \Delta_{sy} \ \Delta_{sz} \right]^T \]

\[ \Delta_m = \left[ \Delta_{mx} \ \Delta_{my} \ \Delta_{mz} \right]^T \]

\[ d^n = \sum_{i=1}^{n} R_i^{-1} \begin{bmatrix} [\Delta a_i] \\ [\Delta a_i] \\ [\Delta a_i] \end{bmatrix} \times p_i \]

III. Error modeling

A. Differential vector of the end-effector

According to the linear error model presented by Veitschegger and Wu, the differential translation vector of the end-effector in the base coordinate due to the kinematics parameter errors in each of the joint coordinates can be expressed as follows:

\[ d^n = \sum_{i=1}^{n} \begin{bmatrix} [\Delta a_i] \\ [\Delta a_i] \\ [\Delta a_i] \end{bmatrix} \times p_i \]

where \( R_i^{-1} \) is the rotational matrix of \( A_i \), \( R_i^{+1} \) is the rotational matrix of \( T_{i-1} \), \( T_i = A_i \cdots A_{i-1}, T_i = I \) , I is an identity matrix, \( p_i = [p_{mx} \ p_{my} \ p_{mz}]^T \) is the positional vector of \( T_i \).

It can be simplified as following:

\[ d^n = \sum_{i=1}^{n} R_i^{-1} \begin{bmatrix} [\Delta a_i] \\ [\Delta a_i] \\ [\Delta a_i] \end{bmatrix} \times p_i \]

B. Error modeling of master slave surgical robot system

The position error of the master-slave surgical robot system can be taken as the difference between the real position and the theory position of the slave manipulator’s end-effector.

Let \( P_{m} \) be the real end-effector position of the master manipulator, the theory end-effector position of the slave manipulator can be written as:

\[ P_s = RKP \]

According to (5)and (6), the real end-effector position of the slave robot corresponding to \( P_{m} \) can expressed as :

\[ P_s = RKP_{m} + d \]

where \( d = [d_{mx} \ d_{my} \ d_{mz}]^T \) is the differential translation vector of the master manipulator, \( d_s = [d_{sx} \ d_{sy} \ d_{sz}]^T \) is the differential translation vector of the slave manipulator.

Then, the differential translation vector of the master slave surgical robot system can be written as :
\[
d = P^*_m - P_s = -RKd_m + d_s
\] (13)

Substitute \( R \), \( K \) into (13), the differential translation vector of the end-effector in the end-effector coordinate frame can be written in component form as:

\[
\begin{align*}
    d_x &= n_x o_x a_x k_x 0 0 d_{max} + d_{sx} \\
    d_y &= n_y o_y a_y k_y 0 0 d_{my} + d_{sy} \\
    d_z &= n_z o_z a_z k_z 0 0 d_{dz} + d_{sz}
\end{align*}
\] (14)

Similarly, the differential rotation vector of the master slave surgical robot system can be written as:

\[
\delta = -R\delta_m + \delta_s
\] (15)

Substitute \( R \) into (15), it can be written in a component form:

\[
\begin{align*}
    \delta_x &= n_x o_x a_x \delta_{mx} + \delta_{sx} \\
    \delta_y &= n_y o_y a_y \delta_{my} + \delta_{sy} \\
    \delta_z &= n_z o_z a_z \delta_{dz} + \delta_{sz}
\end{align*}
\] (16)

where: \( \delta_m = [\delta_{mx}\ \delta_{my}\ \delta_{dz}]^T \) is the differential rotation vector of the master manipulator, \( \delta_s = [\delta_{sx}\ \delta_{sy}\ \delta_{sz}]^T \) is the differential rotation vector of the slave manipulator.

C. Error bound of master salve surgical robot system

Taking the norm on both sides of (13), we have:

\[
|d| = \| -RKd_m - d_s \| \leq \|RKd_m\| + \|d_s\| \] (17)

Since \( R \) is an Unit orthogonal matrix, \( \|RKd_m\| = \|Kd_m\| \), thus (17) can be written as:

\[
|d| = \|Kd_m\| + \|d_s\| \leq \|K\| \|d_m\| + \|d_s\| \] (18)

Therefore, the position error bound of the master slave surgical robot system are defined as follows:

\[
d_{max} = \|K\|\|d_m\| + \|d_s\| \] (19)

If \( K \) is unit matrix, then:

\[
d_{max} = \|d_m\| + \|d_s\| \] (20)

Similarly, taking the norm on both sides of (15), we have:

\[
|\delta| = \|\delta_m - \delta_s\| \leq \|\delta_m\| + \|\delta_s\| \] (21)

The orientation position error bound of the master slave surgical robot system are given as follows:

\[
\delta_{max} = \|\delta_m\| + \|\delta_s\| \] (22)

IV. Simulation

In this section, a simulation study of the proposed error modeling methodology was applied to a surgical robot system, which is composed of the daVinci® as the slave manipulator and the PHANTOM® as the master manipulator. To perform this simulation study, a computer program has been developed in MATLAB. The inputs to the program are the nominal kinematics parameters and there error bound.

![Flowchart of the simulation program](image)

The flowchart for the simulation is shown in Figure 2, and summarized as follows:

Step1. Read nominal kinematics parameters of the master slave surgical robot system and there error bounds.

Step2. Calculate the homogenous transformation matrix between the joint coordinate \( i \) and the end-effector coordinates and the homogenous transformation matrix between the base coordinates and the joint coordinate \( i \). Generate the error model of the master salve surgical robot system.

Step3. Divide the workspace of the master manipulator into \( N \times N \times N \) grids and calculate the end-effector position...
and orientation of the master manipulator at each grid node using the forward kinematics.

Step 4. According to the linear mapping relationship between the master manipulator and the manipulator robot, solve the joint angle of the slave manipulator using the inverse kinematics.

Step 5. Calculate the error bound of the master slave surgical robot system at all grid nodes, and plot the error surface of the cross section.

The nominal kinematics parameters of the PHANTOM® and the daVinci® are listed in Table 1 and Table 2, respectively. The kinematics parameters errors will vary in different regions of the robot workspace due to the geometric errors and the non-geometric errors [11]. For simplicity, assume that the maximum error bounds for angle and length are ±100 µrad and ±10µm, respectively.

<table>
<thead>
<tr>
<th>link</th>
<th>α (rad)</th>
<th>a (mm)</th>
<th>θ (rad)</th>
<th>d (mm)</th>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>θ₁</td>
<td>l₁</td>
</tr>
<tr>
<td>2</td>
<td>-π/2</td>
<td>0</td>
<td>θ₂</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>l₂</td>
<td>θ₁ - θ₁</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-π/2</td>
<td>0</td>
<td>θ₁</td>
<td>-l₁</td>
</tr>
<tr>
<td>5</td>
<td>π/2</td>
<td>0</td>
<td>θ₁</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>π/2</td>
<td>0</td>
<td>θ₁</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE 1. Kinematics parameters for PHANTOM**

<table>
<thead>
<tr>
<th>link</th>
<th>α (rad)</th>
<th>a (mm)</th>
<th>θ (rad)</th>
<th>d (mm)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>0</td>
<td>l₁</td>
<td>0</td>
<td>t₁</td>
</tr>
<tr>
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<td>θ₂</td>
<td>h₂</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>l₁</td>
<td>θ₁</td>
<td>h₁</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>l₁</td>
<td>θ₁ - θ₂/2</td>
<td>h₁+h₂</td>
</tr>
<tr>
<td>5</td>
<td>-π/2</td>
<td>0</td>
<td>θ₁</td>
<td>l₃</td>
</tr>
<tr>
<td>6</td>
<td>π/2</td>
<td>0</td>
<td>θ₁</td>
<td>h₂</td>
</tr>
<tr>
<td>7</td>
<td>-π/2</td>
<td>0</td>
<td>θ₁ - θ/2</td>
<td>l₅</td>
</tr>
<tr>
<td>8</td>
<td>-π/2</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>9</td>
<td>π/2</td>
<td>0</td>
<td>θ₁</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE 2. Kinematics parameters for daVinci**

Figure 3 shows the simulation result of the end-effector position error of the master-slave surgical robot system in the x direction, which falls in the range of 0.1983mm to 0.2385mm. It can be seen that the position error in the x direction is strongly affected by the z axis value, and not much affected by the x axis value. The position error in the X direction is increasing with the z axis value.

Figure 4 shows the simulation result of the end-effector position error of the master-slave surgical robot system in the y direction, which is in the range of 1.0329mm to 1.0984mm. It is shown that the position error in the y direction is affected by both the x and the z axis values.

Figure 5 shows the simulation result of the end-effector position error in the z direction, which is in the range of 0.6826mm to 0.7415mm. It can be seen that the position error in the z direction is strongly affected by the x axis value, and not much affected by the z axis value. The position error in the z direction is increasing with the x axis value.

These results show that the position error will vary in different regions of the robot workspace, and the position error in the z direction and the y directions are mostly bigger than the position error in the x.

**V. Conclusions**

The expressions of the differential translation vector and the differential rotation vector of the end-effector presented by Veitschegger and Wu have been simplified. According to these new expressions, the position and orientation error model of the master-slave surgical robot systems as a function of the robot kinematics parameters...
errors has been presented based on the Khalil and Kleinfinger notation. An obvious way to show the error distribution of the robot manipulator has been proposed. A computer program has been developed in MATLAB which can calculate the error model by using of symbolic computation and perform the accuracy analysis on any master slave robot systems. This error model and software are applied to a master slave surgical robot system. The position error in its workspace cross section (XOZ) has been plotted as 3D surface graph. As a result, this research provides an essential basis for the accuracy analysis and compensation of master slave robot manipulators.

References