Stabilizing Control of Personal Mobility with a Spherical Wheel Composed of Serial Kinematic Chain

Sungsuk Ok* 
University of Tokyo 
Tokyo, Japan

Yoshihiko Nakamura† 
University of Tokyo 
Tokyo, Japan

Abstract—We propose a new personal mobility of wheeled inverted pendulum type. One feature of the mobility is consisting of one spherical wheel and a boarding plate for a rider. The shape of a wheel is similar to a basketball and traveling and steering are achieved by simply changing the relative position of the rider’s center of gravity on the boarding plate. Another feature is that there are three rotation axises for driving the spherical wheel. The rotating axises for spherical wheel are intersecting each other. We call this mechanism a spherical wheel with three-intersecting-axes driving system. This paper show the detail structure and mechanism of new mobility, linear quadratic regulator (LQR) controller system for stabilization and some results of the simulation and experiments using designed controller.

Keywords: personal mobility, spherical wheel, inverted pendulum

I. Introduction

Recent years, many studies for wheeled-type mobility have been reported. Each of them has its advantage and disadvantage. Among others, wheeled inverted pendulum type mobility draw public attention. Thus far, a number of studies concerning two-wheeled self-balancing mechanisms have been conducted [1], [2], [3], [4], [5], [6]. M. Sasaki et al. [7], [1], [2] presented the estimation of posture and control algorithm of riding type wheeled mobile platform by using adaptive observer. Johan et al. [8] designed and controlled the mobile robot of wheeled inverted pendulum type by implemented high accuracy velocity estimation algorithm using analog encoder signals. Nakajima et al. [9] proposed a new mechanism of inverted pendulum type mobile robot using a unicycle. One of the most famous two-wheeled inverted pendulum type mobility is Segway designed and controlled in by [17]. The Segway of two-wheeled self-balancing mobility has been a popular platform in recent years. It is a new type of personal vehicle that has only two coaxially mounted wheels and a steering bar, and occupying small space. Its method of propulsion is a kind of wheeled inverted pendulum that can be balanced by the driving force of each of the two wheels to stabilize the whole system including the rider. Forward and backward movement is conducted by changing rider’s the position of center of gravity, and mobility is steered by rotating steering bar. The direction of steering is achieved by differential angular velocity of two wheels. Therefore, it is impossible for Segway to move omni-directional movement.

In 2005, Ralph L. Hollis group introduced the Ballbot, a dynamically stable mobile robot moving on a single spherical wheel [10], [11] and popularized in [12], [13], [14]. The single spherical wheel provides omni-directional movement making the ballbot more suitable for navigation in human environments with constrained spaces. T. Endo et al. [15] developed an omni directional vehicle on a spherical tire. They selected a basketball as a spherical tire and the vehicle moves by rotating a spherical tire. Omni-directional vehicles are useful for various environments. For instance, they are useful to change direction and move in the narrow spaces. M. Kumagai et al. [16] also developed omni-directional mobile robot of 2-dimensional wheeled inverted pendulum type by utilizing sphere wheel.

In this research, we propose another type of a personal vehicle that consists only of one spherical wheel and a standing plate for the rider, as shown in Fig.1. One feature is that our new mobility is also wheeled inverted pendulum type mobility like Segway and other studies. For above reason, the first object is to stabilize attitude and velocity of the mobility and spherical wheel. The other feature is that our new mobility has three rotation axises rolling the spherical wheel referenced in Fig.2. Although our mobility uses one spherical wheel as a Ballbot and Basket Ball Rider (BBR) [16], it has one more driving axis rotating spherical wheel. The extra actuators can be utilize to
perform other works satisfying attitude control of mobility.

In this paper, the mechanical design and concept are given in Chapter 2. Chapter 3 provide a simplified dynamic modeling, with attitude stabilization. Chapter 4 describe simulation results using Matlab and Open Dynamics Library[18]. Section 5 present forward movement experiments with attitude stabilizing control of mobility.

II. Design of the Mobility

A. Driving Mechanism of Spherical Wheel

Basket Ball Rider[15] had a pair of drive and opposing passive rollers for each of the orthogonal motion directions. Ballbot had four rollers actuated with individual DC servomotors which exert pure torques on the ball. However, above two systems utilize friction between sphere and driving rollers to transmit torque to spherical wheel. Therefore, it was difficult to design friction model and transmit adequate torque to wheel when the slip occurs between wheel and actuated rollers. To avoid these problems, our new mobility has three serial three links connected to the spherical wheel, directly. Three rotation axis are designed to drive a spherical wheel in each joint of link. The three actuated links exert pure torque on the wheel with no slip. Fig. 2 illustrate the simplified driving mechanism of spherical wheel by using three serial links intersecting on the center of spherical wheel.

Fig. 2. Driving mechanism of spherical wheel

Let us define a coordinate system of each link with \( x \), \( y \) and \( z \) axes as shown in Fig. 2. The origin of each link coordinator is placed at the joint of each link. We use "pitch" or "pitching" as an angular motion relative to \( x \) axis, "pitch" or "pitching" as an angular motion relative to \( y \) axis, and "yaw" or "yawing" relative to \( z \) axis. Inertial coordinator is defined with \( X \), \( Y \) and \( Z \) on the center of the spherical wheel at first time. In the spherical wheel driving mechanism, the mobility could be regarded as wheel and pendulum of wheeled inverted pendulum in longitudinal plane. \( \phi_{\text{pitch}} \) is the angle of wheel and \( \theta_{\text{pitch}} \) is the angle of pendulum in the longitudinal plane (YZ plane in Fig. 2). As the same way, spherical wheel including pitch link and mobility are simplified to the wheeled inverted pendulum type in lateral plane. In lateral plane (XZ plane in Fig. 2), spherical wheel including pitch link can be simplified as a rotating wheel and the others are regarded pendulum in the wheeled inverted pendulum model as shown in Fig. 3.

B. Mechanical design

This chapter present about detail mechanical structure and specifications of mobility. The specification of mobility is shown in Tab 1. As mentioned last section, our new mobility could be simplified to wheeled inverted pendulum. Therefore, it is possible to estimate the needed torque roughly to invert the mobility by using Eq.(1).

\[
T = mglsin\theta = Fr
\]

In above equation, \( m \) is mass of mobility and boarding person. \( l \) is distance from center of wheel to COG of mobility including boarding person. \( F \) is translational force to be add to the center of sphere. \( r \) is radius of spherical wheel. \( g \) is gravity acceleration, \( \theta \) is inclination angle relative to vertical axis of inertial coordinator. The mass and COG position are changing when a person is getting on the mobility. Obviously, high torque is needed to invert mobility when person is riding on the mobility than usual wheeled

<table>
<thead>
<tr>
<th>value of parameters</th>
<th>actuator</th>
<th>the maximum continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200[W] brushless motor</td>
<td>0.12 [Nm]</td>
</tr>
<tr>
<td>motor torque</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>gear ratio</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>radius of wheel</td>
<td>0.15 [m]</td>
<td></td>
</tr>
<tr>
<td>traveling speed</td>
<td>about 1[ms]</td>
<td></td>
</tr>
</tbody>
</table>
Inverted pendulum. For those reason, we use two motors to rotate one joint. In general, it is difficult to synchronize velocity and position of two motors connected to one joint. However, when the control target is torque of motors, it does not happen so sensitive property among two motors. We have no regard for the problem in the case of controlling same torque of two motors. There are several merits to adapt double-motor system. It can generate two-times high torque than one motor and reduce the backlash effect by sawtooth of pulley. The specification of mobility and hardware components is shown in Tab. 1. By utilizing double-motor system, high torque can be generated to invert mobility. In each rotation axis, the torque of 24[Nm] could be generated according to the Tab.I and Eq.(1). It is possible for above torque to invert the about 10[deg] inclined mobility or 2.5[deg] inclined mobility and rider when a person of 60[kg] weight is getting on the mobility. The center of gravity position is assumed to 0.6[m] from ground when a person is riding on the mobility.

There are some space in the spherical wheel because we designed spherical wheel of 30[cm] diameter. To use the space effectively, two motor and drivers is installed for driving pitch angle. The mechanical structure of spherical wheel is shown in Fig.4. The pitch rotate angle of spherical wheel is related to the forward / backward traveling as it is shown in Fig.5. In the same way, roll rotate angle is connected to the left / right movement of mobility as shown in Fig.6. The double motor driving system is also installed to rotate roll joint of spherical wheel. The two rotation link can generate same torque performance and roadability. The mechanical structure of new mobility system is shown in Fig. 4. When the roll joint rotate, the pitch link and spherical wheel are rotate about $y$ axis in Fig. 2. From serial link point of view, the spherical wheel is the first link and pitch link is connected to the spherical wheel as second link. And roll link is connected to pitch link as the third link. Yaw link is the last serial link of mobility and also the part of riding on the mobility. The role of yaw rotation angle is to avoid driving limit region of roll axis as shown in Fig.4. There is no limit angle and constrained conditions when the pitch joint rotate the sphere wheel. However, the part of pitch link is connected is not a perfect sphere surface. The part can not contact with ground and not be utilized as footprint. Therefore, the part of not a sphere surface have to be avoided before contacting with ground. For that reason, we have designed the yaw rotation of extra axis. In this paper, it will not be described about yaw angle control to avoid limit area of not a perfect sphere.

III. Posture Stabilization Control

A. Simplified dynamics modeling of 2-dimensional wheeled inverted pendulum

In this chapter, we present linearized motion equation in longitudinal and lateral plane, which are the bases for simple linear feedback controllers. As is mentioned in Chapter
modeling of the new mobility is simplified as a wheeled inverted pendulum in lateral and longitudinal plane. We use linearized model and ignored coupling terms among pitching, rolling and yawing rotation for the time being. The following assumptions are made for simplified modeling.

- there is no slip between the spherical wheel and the floor in the longitudinal and lateral plane, respectively.
- the motion in the longitudinal and lateral plane is decoupled.
- the motion equation in these two planes are identical, the parameters are different.

Euler-Lagrange motion equation is used to derive the dynamic motion equations of each plane. Considering the schematic figure of the wheeled pendulum shown in Fig.2 and Fig.3, the angle between the boarding plate and the vertical axis at inertial coordinator is referred as the pendulum angle \( \theta \), and the rotation angle of the spherical wheel is referred as the rotation angle \( \phi \) of the wheel.

\[
\begin{bmatrix}
I_p & m_plr_w \cos \theta & 0 & 0 \\
-mlr_w \sin \theta & lr_w + m_w r_w^2 + m_p r_w^2 & 0 & 0 \\
0 & -m_pg \sin \theta & \frac{2}{r} & \frac{m_wgl}{r} \\
0 & -m_plr_w \sin \theta & \frac{1}{r} & \frac{m_pgl}{r}
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta} \\
\ddot{\phi}
\end{bmatrix} + \begin{bmatrix}
\tau \\
\tau
\end{bmatrix}
\tag{2}
\]

where \( I_p \) is the moment of inertia of the pendulum with respect to the pivot, \( m_p \) is the mass of pendulum, \( l \) is the distance from the pivot to the center of gravity of the pendulum. \( m_w, r_w \) and \( I_w \) are the mass, radius and moment of inertia of the wheel assembly respectively and \( g = 9.81 \text{ m/s}^2 \) is the acceleration due to gravity.

The stabilizing control strategy described in next section requires a linear state space model. Introducing the state vector \( x = \begin{bmatrix} \theta & \dot{\theta} & \phi & \dot{\phi} \end{bmatrix}^T \), and linearizing the model Eq.(2) around \( x = \begin{bmatrix} 0 & 0 & \phi & \dot{\phi} \end{bmatrix}^T \) gives following Eq.(3), (4).

In other words, Eq.(2) has small range so we can approximate as \( \sin \theta \approx \theta, \cos \theta \approx 1 \) and \( \theta^2 \approx 0 \) for boarding plate inclines very slowly when we assume \( \theta \ll 1 \text{ rad} \). Through this approximating process, followed linear state equation can be approximated when we define Eq. (2) as a state variable \( x = \begin{bmatrix} \theta & \dot{\theta} & \phi & \dot{\phi} \end{bmatrix}^T \).

\[
\dot{x} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\frac{\alpha \beta}{\alpha \beta \gamma} & 0 & 0 & 1 \\
-\frac{\gamma}{\alpha \beta \gamma} & 0 & 0 & 0 \\
\frac{\beta}{\alpha \beta \gamma} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\theta & \dot{\theta} & \phi & \dot{\phi}
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & 0
\end{bmatrix}
\tag{3}
\]

\[
y = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\theta & \dot{\theta} & \phi & \dot{\phi}
\end{bmatrix} \tag{4}
\]

where

\[
\alpha = I_p + m_w r_w^2 + m_p r_w^2, \quad \beta = I_p, \quad \gamma = m_p l r_w, \quad \delta = m_p g l
\tag{5}
\]

The input of the Eq.(2) is the torque applying to motor drivers, \( \tau \). In the next section, it will be presented about the needed torque to stabilize attitude of mobility.

**B. Stabilization controller design of mobility**

There could be many methods to determine advantages of control from state feedback control. One of methods to determine the advantage of a control systematically is LQR(Linear Quadratic Regulator) which is a sort of optimal control. First, quality factor which becomes the standard of the best control is defined as followed

\[
J = \int_0^\infty (x^T Q x + u^T R u) dt
\tag{6}
\]

By applying proper weight matrix \( Q \) and \( R \), state feedback control gain matrix \( K \) is obtained. At this time, weight matrix \( K \) is can be determined through minute investigating on results of repeated simulation which shows the analysis with the limit of control input. In wheeled inverted pendulum model, state space variable is the matrix of \( 4 \times 1 \) as presented in Eq. (2), (3) and (4). Therefore, \( Q \) is the matrix of \( 4 \times 4 \) and \( R \) is \( 1 \times 1 \) matrix.

The spherical wheel angle \( \phi \) of mobility is measured by incremental encoder of each joint. And the angular velocity of spherical wheel can be obtained by differentiating \( \phi \).
with respect to time. The angle $\theta$ and angular velocity $\dot{\theta}$ of boarding plate is measured by 3-axis gyro sensor. The control loop diagram which we designed by utilizing LQR controller is shown in Fig.7. This feedback loop using LQR controller is for one plane of wheeled inverted pendulum. Therefore, we need to design the controller for the lateral plane motion (XZ plane in Fig.2, 3). In this paper, we do not consider for the coupling terms between pitching and rolling motion. It is applied each LQR controller in longitudinal and lateral planes, respectively. The blocked part by a dashed line is state space equation of wheeled inverted pendulum model in Fig.7. The control signal $u$ is obtained by difference of state reference $x_{ref}$ and current value $x$ as Eq.(10)

$$\dot{x} = \begin{bmatrix} A & -BK \end{bmatrix} x + BKx_{ref}$$ (7) 

$$K = R^{-1}B^TP$$ (8) 

$$u = K(x_{ref} - x)$$ (9) 

$$= K \begin{bmatrix} \theta_{ref} - \theta \\ \dot{\theta}_{ref} - \dot{\theta} \\ \phi_{ref} - \phi \\ \dot{\phi}_{ref} - \dot{\phi} \end{bmatrix}$$ (10) 

$$PA + A^TP - PBR^{-1}B^TP + Q = 0$$ (11)

Since sensing of all states of the robot are available by sensors and incremental encoders, state feedback can be conveniently used to stabilize the system. The control law is then given by Eq.(10). We know the system matrix of state space model of wheeled inverted pendulum by CAD program. The $P$ in Eq.(8) is calculated by solving Riccati algebraic equation (Eq.11) of state space model of mobility in each plane. The matrix $P$ is definite symmetry matrix of Riccati algebraic equation and could be determined uniquely when the system matrix $A$, $B$ and weighting matrix $Q$, $R$ is determined. There are many method to solve Riccati algebraic equation. Among them, we used Potter’s method and calculated the gain of controller. In Matlab, there is very useful command to solve Riccati algebraic equation. However, the mass, inertia and other parameters of pendulum model is changing when the person is getting on the mobility. For the reason, the real-time solver of pendulum model is implemented to modify the feedback gain when the person is riding on the mobility. The state feedback gain $K$(longitudinal plane) by parameters of only mobility is given as:

$$K = \begin{bmatrix} -236.8502 & -58.3549 & -20.000 & -16.5696 \end{bmatrix}$$

The stabilizing control simulation result of mobility in longitudinal plane is shown in Fig.8. It is confirmed that boarding plate comes back to the balanced state within 2 seconds through approximately 0.15[rad] of overshoot from the beginning inclination of 0[rad]. The same control law is applied to the rolling rotation in the case of lateral plane (XZ plane in Fig.2). Of course, the state feedback gain $K$ is calculated by utilizing rolling motion equation of lateral plane. Also the parameters and variables are calculated by CAD program. The value of feedback gain $K$ for stabilization of lateral plane (roll or rolling) is given as:

$$K = \begin{bmatrix} -239.0861 & -53.6891 & -25.000 & -18.7041 \end{bmatrix}$$

The angles of mobility and wheel becomes approximately 0.2[rad] to opposite direction and astringents to 0 as the mobility becomes balanced by Fig.9. As above simulation
result, the attitude stabilization of mobility was confirmed by designed LQR controller. Although, designed controller is working well, in simulation, there are several problems to implement to real mechanical system. In Matlab simulation, the plant model was designed by state space model, and the gain is calculated by Riccati equation. In this case, the performance of Matlab simulation is revaluated because linearized plant model is utilized to confirm designed controller. The next section, it will be presented graphical simulation using the dynamics library and CAD data.

C. Simulation with dynamics library

We have done stabilization simulation by using inertia, mass, shape data of CAD program. After we transferred the shape data of CAD file to the DirectX file, and executed simulation using Open Dynamics Library[18]. The center of gravity position of the link is referenced by CAD program. Fig.12, 13 show the scene of the simulation which was applied by LQR controller. The simulation results is shown in Fig.10, 11. In Fig.10, it can be find that mobility comes back to the balanced position within 2 seconds through approximately 0.5 [rad] overshoot from the beginning inclination angle 0.2 [rad] ( θ_{pitch} = 0.21 [rad] ). At this time, angular velocity of mobility becomes approximately 2.5 [rad/s] to opposite direction and astringents to 0 as the boarding plate becomes balanced. On the other hand, wheel rotate approximately 6 [rad/s] to recover the balance then rotate back slowly to the original point as the boarding plate recovers the balance. The stabilization simulation of roll is conducted as initial angle 0 [rad] ( θ_{roll} = 0 [rad] ). As you see in Fig.11, the angles of boarding plate and wheel come to zero. Also, angular velocities of wheel and boarding plate come to zero in 2 seconds. We assume that the ripples of the roll stabilization are affected by pitch motion while pitch controller stabilize the posture of boarding plate in longitudinal plane.

IV. Experiment

A. Stand up and attitude stabilizing control

Experiments with stand up and balance stabilizing control were conducted to confirm the effectiveness of the designed linear quadratic regulator controller using our new mobility. In the experiment, the mobility stand up from the initial state inclined about 3 [deg] (about 0.05 [rad]) relative to vertical axis and stabilized the posture of mobility. The illustration about stand up motion is shown in Fig.15. The data obtained from gyro sensor installed to mobility is shown in Fig.16. The input command of mobility was zero position and velocity of spherical wheel when the standing up experiment is done. Stand up experiment conducted
in this time was done to small inclined angle and person did not get on the mobility. The additional experiment is needed to give person ride on mobility.

After stand up from initial state, attitude of mobility was stabilized by designed controller. The attitude of the mobility in two orthogonal directions are controlled separately. The purpose of control was to maintain inclination angle of mobility to be zero and to maintain position of spherical wheel. The result of attitude control is shown in Fig.16. As shown in Fig.17, the time to stand up was under 1[sec] and very fast. The controller gain was same in experiments to stand up and stabilize the mobility. Although, the same controller gain was used, the reason why the mobility could stand up was the small initial inclined angle. To stand up the mobility when a person is riding, additional experiments and control algorithm will be necessary.

B. Forward movement

The experiment result of wheel velocity control is shown in Fig.17. The reference of wheel velocity was 0.4[rad/s]. The snap shot of wheel velocity control experiment is shown in Fig.14. The snap shot was taken every 2 second unit. We had done one more experiment as the stabilizing performance test. Fig.18, 19 are the results of experiments with the reference of zero wheel position and velocity. Although, the adequate disturbance was given back and forth
by hand during the mobility was stabilized, the wheel came back to the initial position.

V. Conclusions

This paper proposed the new mechanism driving spherical wheel for omni-directional movement. The sphere can be positioned in the plane surface at any point and at any time because of the symmetry characteristic of sphere. We designed omni-directionable mobility based on the above characteristic. However, the mobility can not keep attitude itself because it has just one sphere wheel. The controller stabilizing its attitude is positively necessary. Our new mobility can be simplified as wheeled inverted pendulum mobile robot in sagital and coronal plane, respectively. The LQR suggested as posture balance controller of the mobility is one sort of state variable feedback controller and has character that enabling it to have advantage of controller systematically while requesting state variable. The state variable which is necessary for the structure of wheeled inverted pendulum system can be obtained easily from sensor, we have adapted linear quadratic regulator (LQR) controller as a attitude stabilizing controller of mobility. In the experiment, mobility stand up and stabilized balanced attitude with respect to vertical axis by itself. However, there are several additional experiment is needed for person to ride on the mobility. First, standing up experiment is necessary on the state of getting on the mobility. Second, in this paper, we did not explain the yaw motion to avoid roll rotation limit region. To realize the omni-directional movement of our mobility, above problem have to be solved. Although, there is several future work for the omni-directional movement, we have stabilized the mobility using spherical wheel composed of serial kinematic chain. It was possible to generate and transmit the high torque to the sphere by using serial kinematic chain design.

Acknowledgement

This work was supported by Nissan Science Foundation (2009-2011) on “Research on Perception, Cognition and Human Body in Driving” (PI: Y. Nakamura).

References