Kinematics of New Parallel Structures with 3 and 4 DOF Using Planar Parallel Modules

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Abstract—The use of simple modules in the development of robotic structures lead to robust solutions adaptable to different applications. The authors propose the use of simple two degrees of freedom modules, which have the actuators on the fixed base, in order to develop a 3-DOF family of parallel structures with constant platform orientation, moving in the Cartesian space and a family of parallel mechanisms with 4-DOF which enable the positioning of the mobile platform in the Cartesian space and its orientation around one axis. The geometrical models for each solution are presented. For each family one configuration has been selected being modeled and its algorithm programmed. Simulation results are shown for trajectories in the Cartesian space.

Keywords: kinematics, parallel modules, simulation

I. Introduction

One of the main advantages of parallel robots refer to their small masses in motion, mainly due to the multiple chains connecting the mobile platform to the base and the positioning of the actuators on the fixed platform. Many industrial applications do not require 6-DOF from the robotic system thus encouraging the development of simple solutions with a smaller number of degrees of freedom. The authors propose the use of planar 2-DOF modules which would position all the actuators on the base of the robot. The different configurations are suitable for applications in various fields, from the manipulation of large masses (in construction) to micro-motions.

Many authors studied the development of parallel robots with 3 and 4 DOF, aiming to improve the performances of the system with respect to specific tasks.

Mourad Karouia and Jacques M. Herve present in [1] a study of the kinematics of parallel spherical manipulators with 3 DOF. Another study of the kinematics is presented by Pashkevich for the 3 DOF robot named Orthoglide[2], whose kinematics behavior is similar to one of a Cartesian manipulator. The kinematics model for a type of parallel hybrid robots with 3 independent kinematic chains is presented in [3]. Zoppi shows his work in the development of the inverse kinematics model of 4 DOF type robots, [4]. Another study of the kinematics model of a group of 4 DOF parallel robots is made in [5].

[6] presents the inverse kinematics model of micro parallel mechanisms with translational modules and a method for designing structures with 3, 4 and 5 passive joints. The study of a new 4SPS and UPU parallel manipulator is shown in [7]. This study contains the complete geometrical and kinematics model. Lukanin proposed in [8] the inverse kinematics model and the determination of the workspace for a 3 SPR parallel robot. Another kinematics model is described in [9] for a 3 – CRR parallel robot.


Zhou developed a new parallel spatial mechanism with 4 leading sliders and presents it’s kinematics model in [12]. Badescu studies in [13] the workspace of a 3 – UPU parallel platform based on the inverse and direct kinematics model. In [14] was designed a new 3 – PUU parallel mechanism whose leading translational actuators are displayed on a radial direction. In [15] was designed a hybrid robot with 3 DOF composed by a parallel mechanism and a pantograph mechanism with the purpose of an increased rigidity and workspace volume. The kinematics and dynamics model was also presented in this paper.


Regarding the microrobots, two types of 3 DOF micro parallel manipulators are described in [19] and [20]. Another 3 DOF parallel manipulator is HANA, described in [21]. This robot is used for industrial manipulators and
micro-motion manipulators. Qinsong developed in [22] a very precise 3 DOF micromanipulator with piezoelectric actuators and elastic joints.


II. Planar 2-DOF parallel modules

A set of simple parallel modules with 2 DOF used for the further development of parallel mechanisms with 3 and 4 degrees of freedom are in the following proposed.

A. First Module

Figure 1 illustrates a planar 2-DOF parallel mechanism with the actuators positioned on the fixed base. The direct geometrical model (DGM) and the inverse geometrical model (IGM) are computed.

For solving the DGM the following parameters are given: \( q_1, q_2, d, r, x_p, y_p \) and the unknowns are: \( X_p, Y_p \). From the kinematic scheme, the following equations yield:

\[
\begin{align*}
X_p &= X_A + x_p = q_1 + r c\phi + x_p \\
Y_p &= Y_A + y_p = r s\phi + y_p
\end{align*}
\]

Knowing that:

\[
\begin{align*}
c\phi &= \frac{q_2 - q_1}{2} - \frac{r}{2} \\
s\phi &= \frac{1}{2r} \sqrt{(2r)^2 - (q_2 - q_1)^2}
\end{align*}
\]

The angle \( \phi \) can be expressed as:

\[
\phi = \tan^{-1} \left( \frac{r(2r)^2 - (q_2 - q_1)^2}{q_2 - q_1} \right)
\]

Based on (1), (2) and (3) the coordinates of the end-effector are:

\[
\begin{align*}
X_p &= \frac{1}{2} (q_1 + q_2) + x_p \\
Y_p &= \frac{1}{2} \sqrt{(2r)^2 - (q_2 - q_1)^2} + y_p
\end{align*}
\]

In order to determine the Inverse Geometrical Model (IGM), besides the geometrical parameters, \( d, r, x_p, y_p \), the coordinates of the TCP are known, namely \( X_p, Y_p \), while trying to determine the coordinates of the actuated joints, \( q_1, q_2 \). From equations (1) to (4) it yields:

\[
\begin{align*}
q_2 &= 2X_p - q_1 - x_p \\
y_p &= \frac{1}{2} \sqrt{(2r)^2 - (q_2 - q_1)^2} + y_p
\end{align*}
\]

Applying simple transformations and introducing the equation (5) in (6) it results:

\[
(Y_p - y_p)^2 = r^2 - \frac{(2X_p - 2q_1 - 2x_p)^2}{4}
\]

From (5) and (7) the expressions of the actuated joints can be written:

\[
\begin{align*}
q_1 &= X_p - x_p - \sqrt{r^2 - (Y_p - y_p)^2} \\
q_2 &= 2 \cdot X_p - q_1 - 2 \cdot x_p
\end{align*}
\]

B. Second Module

Figure 2 illustrates the second proposed planar 2-DOF parallel mechanism.

Based on figure 3, using the geometrical parameters, the following equations result:
\[ X_P = a S \phi + b \]  
\[ Y_P = q_1 \]  
\[ (9) \]

From the kinematic scheme results:
\[ C \phi = \frac{q_2 - q_1}{a} \]
\[ S \phi = \frac{1}{2a} \sqrt{a^2 - (q_2 - q_1)^2} \]  
\[ (11) \]

The expression of the angle \( \phi \) results from (11):
\[ \phi = \tan^{-1}\left(\frac{1}{2} \sqrt{a^2 - (q_2 - q_1)^2} \cdot q_2 - q_1\right) \]  
\[ (12) \]

Using the geometrical parameters \( a, b \) and the equations (9) – (12) the coordinates of the end-effector can be easily expressed as follows:
\[ \begin{aligned}
X_P &= b + \sqrt{a^2 - (q_2 - q_1)^2} \\
Y_P &= q_1 \\
\end{aligned} \]  
\[ (13) \]

Based on the (9)-(13) the position equations for the mechanisms can be written in implicit form:
\[ \begin{aligned}
 f_1(X_P, Y_P, Z_P, q_1, q_2) &= X_P - b - \sqrt{a^2 - (q_2 - q_1)^2} = 0 \\
 f_2(X_P, Y_P, Z_P, q_1, q_2) &= Y_P - q_1 = 0 \\
\end{aligned} \]  
\[ (14) \]

Based on (14) the equations of the IGM can be written as follows:
\[ \begin{aligned}
q_1 &= Y_P \\
q_2 &= q_1 + \sqrt{a^2 - (X_P - b)^2} \\
\end{aligned} \]  
\[ (15) \]

III. Parallel mechanisms with constant platform orientation using the planar modules

The authors will present a family of 3-DOF parallel mechanisms achieved by using the planar modules described before.

A. The initial structure

The mechanism presented in figure 3 has two kinematic chains and linear actuators on levels 1 and 3. In order to improve the performance of the robot the task is to eliminate the translational actuated joint on level 3. Based on the kinematic scheme in figure 3, the following equations can be written:
\[ \begin{aligned}
X_P &= q_1 \\
Y_P &= q_1 C \theta_1 + a + b \\
Z_P &= h - q_3 S \theta_1 \\
\end{aligned} \]  
\[ (16) \]

Using (15) – (19) the position equations of the mechanism can be written as follows:
\[ \begin{aligned}
 X_P &= r_2 C \theta_2 + R_2 C \phi_2 \\
 Y_P &= q_2 + b \\
 Z_P &= h - r_3 S \theta_2 - R_2 S \phi_2 \\
\end{aligned} \]  
\[ (17) \]
B. Parallel mechanism with constant platform orientation using the planar module 1

Figure 4 illustrates a solution developed using module 1, having linear actuated joints and passive rotational joints.

Based on the equations (1)-(8) and the geometrical parameters in figure 4, the following equations can be written:

\[
\begin{align*}
X_p &= q_1^* \\
Y_p &= q_3 C \theta_1 + a + b \\
Z_p &= h - q_3 S \theta_1 \\
X_p &= r_2 c \theta_2 + R_2 c \phi_2 \\
Y_p &= q_2 + b \\
Z_p &= h - r_2 s \theta_2 - R_2 s \phi_2
\end{align*}
\]  

(21)

(22)

Having the geometrical parameters \( d_1 = d/2, R_1, r_2, e_1, R_2 \) and the given coordinates for the actuated joints \( q_1, q_2, q_3 \), the DGM has to be determined.

\[
q_3^* = \frac{1}{2}\sqrt{(2R_1)^2 - (q_3 - q_1)^2} + e_1
\]  

(23)

\[
q_1^* = \frac{1}{2}(q_1 + q_3) + \frac{d}{2}
\]  

(24)

\[
\begin{align*}
q_3 c \theta_1 &= Y_p - a - b \\
q_3 s \theta_1 &= h - Z_p
\end{align*}
\]  

(25)

Using (21)-(25) it results the system of equations that characterizes the end-effector coordinates:

\[
\begin{align*}
X_p &= q_1^* - \frac{1}{2}(q_1 + q_3) + \frac{d}{2} \\
Y_p &= q_2 + b \\
Z_p &= h - \sqrt{(q_3^*)^2 - (Y_p - a - b)^2}
\end{align*}
\]  

(26)

Based on the (21) to (26) the implicit position equations for the mechanism:

\[
\begin{align*}
f_1(X_p, Y_p, Z_p, q_1, q_2, q_3) &= X_p - \frac{1}{2}(q_1 + q_3) - \frac{d}{2} = 0 \\
f_2(X_p, Y_p, Z_p, q_1, q_2, q_3) &= Y_p - q_2 - b = 0 \\
f_3(X_p, Y_p, Z_p, q_1, q_2, q_3) &= (Z_p - h)^2 - \left[\frac{1}{2}\sqrt{(2R_1)^2 - (q_3 - q_1)^2} + e_1\right]^2 + (Y_p - a - b)^2 = 0
\end{align*}
\]  

(27)

For the IGM the equations can be written based on (23)-(27):

\[
\begin{align*}
q_1 &= X_p - \frac{d}{2} - \sqrt{R_1^2 - \left[q_3^* - Y_p\right]^2} \\
q_2 &= Y_p - b \\
q_3 &= q_1 + 2\sqrt{R_1^2 - \left[q_3^* - Y_p\right]^2}
\end{align*}
\]  

(28)

C. Parallel mechanism with constant platform orientation using the planar module 2

A very simple and compact solution derived from the mechanism presented in figure 3 which uses the second planar module, is presented in figure 5.

Using the equations (9)-(15) of the second planar module and the geometrical parameters in figure 5, the following equations can be written:

\[
\begin{align*}
X_p &= q_1 \\
Y_p &= q_3 + d_1 C \theta_1 + a + b \\
Z_p &= h - q_3 + d_1 S \theta_1 \\
X_p &= r_2 C \theta_2 + R_2 C \phi \\
Y_p &= q_2 + b \\
Z_p &= h - r_2 S \theta_2 - R_2 S \phi
\end{align*}
\]  

(29)

(30)

where

\[
q_3^* = \sqrt{R_1^2 - (q_3 - q_1)^2}
\]  

(31)
Using (29)-(31) the equations of the DGM are obtained as follows:

\[
\begin{align*}
X_p &= q_1 \\
Y_p &= q_2 + b \\
Z_p &= h - \sqrt{(q_1 + d_1)^2} - (q_2 - a)^2
\end{align*}
\] (32)

In the same manner the equations of the IGM result:

\[
\begin{align*}
q_1 &= X_p \\
q_2 &= Y_p - b \\
q_3 &= X_p + \sqrt{R^2 - \left(\sqrt{(Y_p - a - b)^2 + (h - Z_p)^2} - d_1\right)^2}
\end{align*}
\] (33)

Based on the (29) to (33) the implicit position equations for the mechanism:

\[
\begin{align*}
f_1(q_1, q_2, q_3, X_p, Y_p, Z_p) &= q_1 - X_p = 0 \\
f_2(q_1, q_2, q_3, X_p, Y_p, Z_p) &= q_2 - Y_p + b = 0 \\
f_3(q_1, q_2, q_3, X_p, Y_p, Z_p) &= (q_3 - q_1)^2 - R^2 + \sqrt{(Y_p - a - b)^2 + (h - Z_p)^2} - d_1^2 = 0
\end{align*}
\] (34)

In order to determine the expression of the \( \theta_2 \) angle, from the equations (29) and (30) it results:

\[
\begin{align*}
R_2 C \theta_2 &= X_p - r_2 C \theta_2 \\
R_2 S \theta_2 &= (h - Z_p) - r_2 S \theta_2
\end{align*}
\] (35)

Squaring the two equations from (35) and adding them leads to the following result:

\[
(2r_2 X_p) C \theta_2 + 2r_2 (h - Z_p) S \theta_2 = X_p^2 - r_2^2 - R^2 + (h - Z_p)^2
\] (36)

This enables the calculation of the \( \theta_2 \) angle:

\[
\theta_2 = \tan^{-1} 2 \left( \frac{1 + \sqrt{a_1^2 + b_2^2 - c_2^2}}{a_2, b_2} \right) = \tan^{-1} \left( \frac{X_p - r_2 C \theta_2}{X_p - r_2 S \theta_2} \right)
\] (37)

Using (37) in (35) the expression of the angle \( \varphi \) is calculated:

\[
\varphi = \tan^{-1} \left( \frac{h - Z_p - r_2 S \theta_2 - X_p - r_2 C \theta_2}{X_p - r_2 C \theta_2} \right)
\] (38)

Based on the kinematic scheme presented in figure 2, another structure can be developed by replacing the rotational joint \( c_8 \) with a translational one. The new solution is presented in figure 6.

D. Parallel mechanism with constant platform orientation using 2 planar modules of the second type

The next kinematic structure proposed by the authors uses two planar modules displayed in vertical configuration, as shown in figure 7.

Fig. 6. Kinematic scheme of a parallel mechanism using the second planar module with translational joints on level 3

Fig. 7. Kinematic scheme of a parallel mechanism using 2 modules of the second type

The actuated joints are denoted with C1, C2 and C3. C2 and C3 ensure the simultaneous displacement of the joints C4 and C8 respectively C5 and C9, while C1 rotates the shaft 1 and with the help of the passive joint C7 it rotates the two horizontal arms. From the kinematic scheme it can be written that:
The equations for the DGM result as follows:

\[
\begin{align*}
X_P &= \frac{b + \sqrt{a^2 - (q_3 - q_2)^2}}{2} C_{q_4} + h, \\
Y_P &= \frac{b + \sqrt{a^2 - (q_3 - q_2)^2}}{2} S_{q_4} + \frac{1}{2} h_2, \\
Z_P &= q_2
\end{align*}
\]

The expression of the angles \( \varphi_0 \) and \( \varphi_2 \) is obtained:

\[
\varphi_0 = \varphi_2 = a \tan \left( \sqrt{a^2 - (r - b)^2}, q_3 - q_2 \right)
\]

For the IGM using the equations (39), (40) and (43) it results the following system:

\[
\begin{align*}
q_1 &= a \tan \left( Y_P - \frac{1}{2} h_2, X_P \right) \\
q_2 &= Z_P + h \\
q_3 &= q_2 + \sqrt{a^2 + (r - b)^2}
\end{align*}
\]

IV. Parallel mechanisms with 4-DOF using the planar modules

The authors propose next several solutions of parallel mechanisms with 4 degrees of freedom which are built by means of the planar modules presented above.

The initial structure is illustrated in figure 8, having two kinematic chains with active joints on levels 1 and 3.

From the kinematic scheme the following equations result:

\[
\begin{align*}
X_A1 &= X_P, \\
Y_A1 &= Y_P, \\
Z_A1 &= Z_P + d \quad (45) \\
X_A2 &= X_P + c \phi, \\
Y_A2 &= Y_P + c \phi, \\
Z_A2 &= Z_P + d \quad (46)
\end{align*}
\]

The DGM equations are determined using the equations (45) – (50), as follows:

\[
\begin{align*}
X_P &= m + c \phi = \sqrt{q_1^2 - q_4^2} + c \phi \phi
\quad (52) \\
Y_P &= q_2 - c \phi \\
Z_P &= q_1 - d
\end{align*}
\]
In order to eliminate the level 3 active joints, the planar module of the second type can be used for each arm of the kinematic scheme in figure 8, resulting the new solution illustrated in figure 9.

Following the same steps, and based on the equations (46)-(50), the implicit functions can be determined:

\[
\begin{align*}
&f_1(q_1, q_2, q_3, X, Y, Z, \rho) = 0 \\
&f_2(q_1, q_2, X, Y, Z, \rho) = 0 \\
&f_3(q_1, q_2, X, Y, Z, \rho) = 0 \\
&f_4(q_1, q_2, X, Y, Z, \rho) = 0
\end{align*}
\]

Using the geometrical parameters and applying several transformations, the equations for the IGM are determined:

\[
\begin{align*}
q_1 &= Z_P + d \\
q_2 &= Y_P + e\rho \\
q_3 &= q_1 - \sqrt{R_1^2 - \left(\sqrt{X_P^2 + Y_P^2} - d_1\right)^2} \\
q_4 &= q_2 - \sqrt{R_2^2 - \left(\sqrt{(X_P - e\rho)^2 + (Z_P + d)^2} - d_2\right)^2}
\end{align*}
\]

In the same manner will result the expression of the coordinates of the end-effector (DGM):

\[
\begin{align*}
X_P &= \sqrt{q_1^2 - q_2^2 + e\rho} \\
Y_P &= q_1 - e\rho \\
Z_P &= q_1 - d \\
\rho &= a \tan 2\left(\frac{X_P^2 + Y_P^2 - Z_P^2}{2XY_P} \right) - a \tan 2\left(\frac{a_P}{b_P}\right)
\end{align*}
\]

V. Implementation and results

The previous algorithms have been implemented in MATLAB and the kinematic schemes were modeled for further simulation.

Some simulation results obtained for one of the 3-DOF structures as well as one with 4-DOF are presented.

In order to obtain a proper simulation of the robots behavior, the MATLAB algorithm contains also the equations for speeds and accelerations, obtained by solving the inverse kinematical model, based on the well known equations:

\[
\begin{align*}
q &= -B^{-1}AX \\
\dot{q} &= -B^{-1}(AX + \dot{A}X + B\dot{q})
\end{align*}
\]

where A and B are the Jacobi matrices obtained by means of the implicit functions.

A. Parallel mechanism with constant platform orientation using the second planar module

For the simulation and algorithm implementation, the solution which has translational joints on the third level has been selected.

The simplified virtual model is presented in figure 10.

The proposed solution uses linear actuators for the active joints 1, 2 and 3 while between elements 4 and 7 respectively 5 and 8 there are only translational relative motion.

The geometrical parameters, based on figure 6, are:

\[
\begin{align*}
d_1 &= 137\text{ mm} \\
d_2 &= 145\text{ mm} \\
a &= b &= 60\text{ mm} \\
r_1 &= 315\text{ mm} \\
h &= 325\text{ mm}
\end{align*}
\]

In the simulation the robot achieved a linear trajectory in space, between two points having the initial and final coordinates, as presented in figure 11:

\[
\begin{align*}
X_i &= 245\text{ mm} \\
Y_i &= 400.84\text{ mm} \\
Z_i &= 121.49\text{ mm} \\
X_f &= 280\text{ mm} \\
Y_f &= 370\text{ mm} \\
Z_f &= 140\text{ mm}
\end{align*}
\]
Fig. 11. Simulation results for a linear trajectory in space achieved by the 3-DOF parallel robot with constant orientation

B. Parallel mechanism with 4-DOF using the second planar module

The simplified virtual model is presented in figure 11.

Based on the kinematic scheme in figure 9, the following geometrical parameters were defined:

\[
\begin{align*}
    d &= 80 \, \text{mm}; \quad r_1 = 315 \, \text{mm}; \quad r_2 = 185 \, \text{mm}; \\
    d_1 &= 141 \, \text{mm}; \quad d_2 = 141 \, \text{mm}; \quad e = 120 \, \text{mm}; \\
    e_1 &= 257 \, \text{mm}; \quad e_2 = 207 \, \text{mm}; \quad h = 325 \, \text{mm}.
\end{align*}
\]

The motion simulation was represented by a linear trajectory in space, with the orientation of the platform. The initial and final coordinates are presented:

\[
\begin{align*}
    X_i &= 150.3 \, \text{mm}; \quad Y_i = 328.73 \, \text{mm}; \quad Z_i = 25 \, \text{mm}; \quad \phi_i = 26^\circ; \\
    X_f &= 210 \, \text{mm}; \quad Y_f = 345 \, \text{mm}; \quad Z_f = 65 \, \text{mm}; \quad \phi_f = 22^\circ.
\end{align*}
\]

The simulation results are presented in figure 13.

V. Conclusions

In this paper the kinematics of new parallel structures with 3 and 4-DOF have been presented. The structures have been developed using simple 2-DOF planar modules and the different configurations cover a large area of applications. The geometrical models for each configuration have been presented. From each family, a configuration has been selected, built and implemented in Matlab. Simulation results were presented, in terms of displacements, speeds and accelerations at the level of the actuated joints for a given spatial trajectory.

The resulting families of 3 and 4 degrees of freedom parallel robots have been submitted for patenting at the Romanian State Office for Patents and Trademarks [28], [29].

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